

# The Macroeconomic Consequences of Government Spending (Re)Allocation

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December 10, 2024

## Abstract

In this paper, we investigate whether changes in the composition of government purchases affect macroeconomic outcomes. To achieve this, we utilize a Factor-augmented Vector Autoregression (FAVAR) model across six spending categories, aiming to estimate the slope and curvature factors that represent the distribution of fiscal spending. We focus primarily on the slope factor, referred to as the reallocation factor, which captures shifts in spending proportions among categories. Our findings suggest that reallocating funds from state and local government consumption to other fiscal components leads to an increase in total federal outlays and output, all without worsening the fiscal deficit. These results highlight the importance of spending composition in influencing fiscal policy outcomes.

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# 1 Introduction

On November 6, 2021, Congress passed the Bipartisan Infrastructure Deal (Infrastructure Investment and Jobs Act), which aims to rebuild America’s roads, bridges, and railways, expand access to clean drinking water, ensure that every American has access to high-speed internet, address the climate crisis, promote environmental justice, and invest in communities that have often been overlooked. Economically, this legislation is expected to help alleviate inflationary pressures and strengthen supply chains by significantly improving our nation’s ports, airports, railways, and roads. It will also create well-paying union jobs and promote sustainable and equitable economic growth. Combined with the President’s Build Back Framework, it is projected to add an average of 1.5 million jobs annually through 2030.

In contrast, on the international front, since Russia’s full-scale invasion of Ukraine in February 2022, Congress has appropriated or otherwise made available nearly \$183 billion for OAR and the broader Ukraine response,<sup>1</sup> with \$130.1 billion obligated and \$86.7 billion disbursed as of September 30, 2024.<sup>2</sup>

While the Infrastructure Bill has received praise, the American electorate continues to grapple with the question of how much of the federal budget should be allocated to international allies at the expense of domestic issues. The government budget represents a strategic allocation of resources across competing areas, such as defense, infrastructure, education, and healthcare. These decisions are fundamental to fiscal policy design, shaping how public spending stimulates economic activity. Adjustments in resource allocation are crucial not only during crises when targeted fiscal responses address immediate challenges, but also in periods of stability when strategic reallocations can strengthen the economy and foster future growth. Therefore, the composition of government expenditure plays a pivotal role in shaping its effects on employment, income, and output.

This paper underscores the importance of shifts in the composition of government spending across various fiscal categories. Recent literature highlights that fiscal multipliers differ by spending category, with the aggregate multiplier for total government purchases approximating a weighted average of the multipliers of individual components (Boehm, 2020). Consequently, the effectiveness of fiscal policy depends not only on its size but also on the composition of the basket of products purchased by the government (Ramey and Shapiro,

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<sup>1</sup>OAR stands for Operation Atlantic Resolve

<sup>2</sup>Source: <https://www.ukraineoversight.gov/Funding>

1998; Muratori et al., 2023). For instance, reallocating spending from state and local government consumption to defense or investment may yield distinct macroeconomic effects. However, the mechanisms underlying these variations remain largely unexplored. This paper fills this gap by examining how allocations across spending components influence macroeconomic outcomes.

An exogenous shock to spending on any individual fiscal component inherently affects expenditures across other components due to underlying correlations. Such shocks can alter the level of government spending (size) or shift the distribution of spending across components, thereby changing the composition of the government’s expenditure basket. For instance, a considerable rise in military expenditures driven by escalating international conflicts, such as those in the Middle East, the war in Ukraine, and other global security threats, might reduce the relative proportion of spending dedicated to education and healthcare. Such shocks to individual spending components are effectively composite shocks, which we decompose into two types: (1) a level shock that affects spending across all categories while keeping their relative proportions constant, and (2) a reallocation shock that shifts spending across components without changing the total expenditure.

Aggregate fiscal shocks, resembling our level shock, allow for changes in proportions and have been widely studied. However, our work centers on analyzing the impact of a reallocation shock involving multiple components. We begin by calculating the share of total spending allocated to each fiscal component. Using a Factor-augmented Vector Auto-regression (FAVAR) model, we estimate two factors that characterize the distribution of spending across these components. The first factor, referred to as the reallocation (or slope) factor, captures shifts in component shares. Since the shares collectively sum to one, a change in one component’s share will necessarily alter one or more of the other shares. In the absence of a contemporaneous change in total spending, this shift in shares directly corresponds to a shift in spending across components. The second factor represents the curvature of the distribution, reflecting changes in its shape. It highlights shifts in the relative significance of smaller versus larger shares within the distribution. A shock to the reallocation factor enables us to assess the macroeconomic implications of reallocating spending from one component to others.

We first briefly explore the effects of a level shock, which serves as a benchmark for comparing our results with the existing literature. For the analysis, total government spending is divided into six distinct components - federal defense and non-defense consumption and

investment as well as state and local government consumption and investment. A level shock results in increased spending across all categories, leading to a corresponding rise in the fiscal deficit and output. These findings align with the existing literature. Furthermore, although the computed level multiplier cannot be directly compared with the government spending multiplier in the literature since it keeps the distribution unchanged contemporaneously, it remains close to reported values. Specifically, we find a level multiplier of 0.56 over a 20-quarter horizon, while Auerbach and Gorodnichenko (2012) reports a multiplier of 0.57 over the same horizon. Similarly, Ramey (2011) estimates multipliers ranging from 0.6 to 1.2, depending on the sample.

A reallocation shock reduces the proportion of spending on the largest component, state and local government consumption, while increasing the shares of the remaining five categories. The largest increase is observed in the share of federal defense consumption, followed by local government investment and federal non-defense investment. This reassignment leads to a steady rise in total spending. At the outset, the fiscal deficit declines as resources transition from consumption to investment, with investment outlays occurring over time. Output exhibits a positive response, with a significant increase observed initially. The reallocation multiplier, which measures the cumulative change in GDP relative to the cumulative change in state and local government consumption, equals -0.11 on impact and declines to -0.8 over a 20-quarter horizon. These early improvements in output and fiscal burden, without subsequent changes in total spending, highlight the potential benefits of reallocating expenditures across components.

This paper provides empirical evidence emphasizing the importance of spending composition in shaping economic outcomes. Adjusting the allocation of spending across components, while holding total spending constant, can generate varied effects on output. Changes in spending patterns over time may partly explain differences in aggregate multipliers across periods, offering policymakers valuable insights for optimizing fiscal strategies within budget constraints.

The rest of the paper is organized as follows: Section 2 reviews the relevant literature that grounds our research. Section 3 covers the data utilized and section 4 details our empirical setup. Section 5 presents and discusses the results, and section 6 concludes.

## 2 Related Literature

This paper is connected to the extensive literature evaluating the factors that influence the effectiveness of fiscal policy. Specifically, this effectiveness is contingent upon various factors, including the state of the economy (Auerbach and Gorodnichenko, 2012; Laumer and Philipps, 2020), the type of financing for fiscal measures (Hagedorn et al., 2019), the response of monetary policy (Woodford, 2011), and country characteristics such as the level of development, exchange rate regime, openness to trade, and public indebtedness (Ilzetzki et al., 2013). Additionally, household heterogeneity (Flynn et al., 2022), firm size heterogeneity (Juarros, 2020), and the composition of fiscal purchases (Bouakez et al., 2020; Boehm, 2020; Muratori et al., 2023) also play significant roles. This paper introduces an empirical framework to compute dynamic responses resulting from shifts in the distribution of spending among various components, contrasting with previous studies (Bouakez et al., 2020; Boehm, 2020; Muratori et al., 2023), which focused on shifts between two specific components.

Furthermore, we contribute to the literature by documenting variations in multipliers across different components of government spending. Auerbach and Gorodnichenko (2012), Ellahie and Ricco (2017), and Laumer and Philipps (2020) report higher multipliers for government investment compared to government consumption, while Perotti (2004) and Boehm (2020) find the opposite, indicating higher multipliers for government consumption. Additionally, Baxter and King (1993), Auerbach and Gorodnichenko (2012), Barro and De Rugy (2013), Ellahie and Ricco (2017), and Laumer and Philipps (2020) report lower multipliers for defense spending, as it does not directly impact the production or utility functions within the economy. Lastly, state and local government spending has been shown to have a higher multiplier than federal government spending components (Ellahie and Ricco, 2017; Laumer and Philipps, 2020). However, Clemens and Miran (2012) finds that state and local expenditures tend to be pro-cyclical due to balanced budget requirements, resulting in multipliers below 1. This finding aligns with Ricardian effects, where households, anticipating future taxes to offset government debt, choose to save rather than spend. At a more granular level, Ellahie and Ricco (2017) demonstrates that state and local government consumption has higher multipliers compared to federal defense and non-defense consumption. In contrast, our findings suggest that a reduction in state and local government consumption spending contributes to increased total spending without raising the fiscal burden, ultimately leading to an increase in output.

### 3 Data

We utilize quarterly data from 1960 Q1 to 2022 Q4 on total government purchases and their components to analyze the effects of reallocating spending among these components. Total government purchases are broadly categorized into consumption expenditures and gross investment. Government consumption expenditures refer to the value of goods and services provided by the government, including areas such as education, law enforcement, and military equipment purchases. When goods and services are offered at a subsidized cost, only the difference between the incurred costs and collected revenues is recorded as government consumption expenditures.

Total government consumption expenditures are further divided into spending by federal and state and local governments. At the federal level, consumption expenditures are categorized into defense and non-defense. Federal defense consumption includes expenditures on military personnel salaries and benefits, operational costs for military bases, equipment procurement, and contracted services essential for defense operations. In contrast, federal non-defense consumption covers salaries and benefits for civilian personnel and goods and services for sectors such as education, health, and transportation, as well as the maintenance of federal facilities like government offices and public health infrastructure. State and local government consumption, similar to federal non-defense consumption, addresses localized needs at the state and community level.

On the other hand, government gross investment is defined as gross fixed capital formation, encompassing investments made by government entities in physical assets, including structures (such as highways), equipment, and intellectual property products. This category is also further disaggregated into federal defense and non-defense investment, as well as state and local government investment. Federal defense investment focuses on defense infrastructure and military equipment, while federal non-defense investment prioritizes public infrastructure, transportation, and health-related initiatives. State and local government investment, akin to federal non-defense investment, centers on localized infrastructure tailored to state and community needs.

The data collected comprises six key components: (1) Federal Defense Consumption Expenditures (DEFEC), (2) Federal Defense Gross Investment (DEFI), (3) Federal Non-Defense Consumption Expenditures (NDEFEC), (4) Federal Non-Defense Gross Investment (NDEFI), (5) State and Local Government Consumption Expenditures (SLGC), and (6) State and

## Local Government Gross Investment (SLGI).

Each component is divided by total government spending to calculate its share and ranked in descending order as follows: (1)  $s_1 = SLGC/G$ , (2)  $s_2 = DEFC/G$ , (3)  $s_3 = SLGI/G$ , (4)  $s_4 = NDEFC/G$ , (5)  $s_5 = DEFI/G$  and (6)  $s_6 = NDEFI/G$ . Figure 1 illustrates the evolution of these shares over time from 1960 to 2022, with shaded gray bars indicating recession periods. The graph demonstrates that SLGC became the dominant component of total government spending after 1970, whereas NDEFI consistently maintained the smallest share throughout the sample period.

During the sample period, the average share of consumption by state and local governments is approximately 44%, while federal defense consumption averages 24%, and federal non-defense consumption expenditures account for 10%. Investment shares are lower, with state and local government investment at 11%, federal defense investment at 7%, and federal non-defense investment at 4%. Figure 1 shows that state and local government consumption is the most volatile, closely followed by federal defense consumption and investment. Federal non-defense consumption and state and local government investment exhibit moderate fluctuations, while federal non-defense investment remains the most stable over time.

Figure 1 indicates that the share of state and local government consumption declines prior to the most recent recessions, suggesting that it may serve as a leading economic indicator. In contrast, for the same recessions, federal defense consumption appears to increase as the share of state and local government consumption decreases. While the abovementioned observations are only correlations, could it be that prioritizing spending on federal defense at the expense of state and local governments contributes to the onset of a recession? This paper aims to explore the shifts in spending among these components and their varying impacts on GDP and potential recessions.

Additionally, the dataset includes information on the fiscal deficit, defined as the difference between government expenditures and receipts as a percentage of GDP, as well as real GDP per capita. More details on data definitions and sources can be found in Appendix A.

## 4 Econometric Framework

### 4.1 Model Specification

Let  $Y_t$  be an  $M \times 1$  vector of observable macroeconomic variables such as government spending, GDP, etc on which we aim to estimate the effects of a shift in the composition of the spending components. Let us assume that there are  $N$  spending categories, denoted as  $G_{1,t}$ ,  $G_{2,t}$ ,  $\dots$ ,  $G_{N,t}$ , which satisfy  $\sum_{i=1}^N G_{i,t} = G_t$  where  $G_t$  is the total government spending.

To estimate shifts in spending across these components, we first calculate the share of each component  $G_{i,t}$  as  $s_{i,t} = \frac{G_{i,t}}{G_t}$ . These individual shares are then successively accumulated to form a series of monotonically increasing cumulative shares:  $g_{1,t} = s_{1,t}$ ,  $g_{2,t} = s_{1,t} + s_{2,t}$ , up to  $g_{N,t} = 1$ . We construct  $\chi_t$  as an  $(N - 1) \times 1$  vector that comprises these cumulative shares, excluding  $g_{N,t}$  since  $g_{N,t} = 1$  for all  $t$ . Using these cumulative shares as our informational series, we estimate two latent factors. The first factor represents the slope, or reallocation effect, which reflects shifts in the distribution of spending across the  $N$  components. The second factor captures the curvature of the distribution, highlighting changes in the relative significance of intermediate categories.<sup>3</sup> The level effect is represented by changes in total spending,  $G$ .

Employing a factor-augmented vector autoregression, as introduced by Bernanke et al. (2005), we assume that the cumulative shares in  $\chi_t$  are related to the latent factors  $f_t$  and observable macroeconomic factors  $Y_t$  as follows:

$$\begin{bmatrix} \chi_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \Lambda^f & \Lambda^Y & 0 & \dots & 0 \\ 0 & I & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} f_t \\ Y_t \\ \vdots \\ f_{t-p+1} \\ Y_{t-p+1} \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \end{bmatrix} \quad (1)$$

$$e_t \sim N(0, R)$$

where  $f_t$  is a  $K \times 1$  vector of latent factors, with  $K = 2$  in our case.  $Y_t$  is an  $M \times 1$  vector of observable macroeconomic variables.  $\Lambda^f$  is an  $(N - 1) \times K$  matrix of factor loadings, and  $\Lambda^Y$  is an  $(N - 1) \times M$  matrix, which we set to 0. We assume that  $\chi_t$  can be written as a

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<sup>3</sup>With only two components, it suffices to focus on the level and the share of one component, as these capture both the average spending (level) and the direction (slope/share) of the expenditure distribution.



linear combination of  $K$  unobservable factors. Since  $Y_t$  is observable,  $I$  is an  $M \times M$  identity matrix.  $e_t$  is an  $(N - 1) \times 1$  vector of normally distributed error terms with mean zero and  $R$  as the diagonal covariance matrix.

Defining  $Z_t = \begin{bmatrix} f_t' & Y_t' \end{bmatrix}'$ , the transition equation follows a VAR(1) process, expressed as:

$$\begin{bmatrix} Z_t \\ \vdots \\ Z_{t-p+1} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_p \\ I & 0 & \dots & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \dots & I & 0 \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ \vdots \\ Z_{t-p} \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \end{bmatrix} \quad (2)$$

$$v_t \sim N(0, Q)$$

where  $\phi_l$  is the  $(K+M) \times (K+M)$   $l$ -lag coefficient matrix for  $l = 1, \dots, p$ .  $v_t$  is an  $(K+M) \times 1$  vector of reduced form errors with a mean of zero and covariance matrix  $Q$ , which we assume to be of full rank.

Given the data outlined earlier,  $N = 6$  comprises federal defense and non-defense consumption and investment, as well as state and local government consumption and gross investment. Additionally,  $M = 3$ , where  $Y_t$  represents a vector consisting of real government spending per capita ( $G_t$ ), fiscal deficit and real GDP per capita. Considering the quarterly frequency of the data, the number of lags is set to  $p = 4$ , in line with the existing literature.

Factor-augmented VAR models are powerful tools for analyzing dynamic systems, offering a flexible framework for capturing time-varying relationships. An alternative approach involves estimating a VAR using factors derived from principal component analysis (PCA). However, PCA factors are often challenging to interpret because their loadings cannot be easily restricted. In contrast, our model restricts the factor loadings, facilitates the interpretation of factors, and models the factors as dynamic processes, effectively capturing the evolving composition of fiscal purchases.

## 4.2 Factor Identification

Before we identify the structural shocks from the reduced form errors  $v_t$  in the VAR specification (2), we need to uniquely identify the factors and their corresponding factor loadings. We begin by arranging the individual shares ( $s_i$ ) in a descending order, from largest to smallest, and then sequentially summing them to obtain the cumulative shares ( $g_i$ ). We designate the first factor ( $f_1$ ) to capture the reallocation effect across these cumulative shares. For this

purpose, we set the loading on  $f_1$  for  $g_1$  as  $\lambda_{1,1}^f = -1$  and impose that the differences between consecutive loadings satisfy  $\lambda_{i,1}^f - \lambda_{i-1,1}^f > 0$  for  $i = 2, \dots, N - 1$ , ensuring a progressively increasing pattern for the loadings on  $f_1$ .

For the curvature factor ( $f_2$ ), we assign the loading for  $g_1$  as  $\lambda_{1,2}^f = 0$ . The restrictions on the factor loadings for  $g_1$  in the measurement equation prevent factor switching. Additionally, because the individual shares are ordered from largest to smallest, we specify that the differences between loadings on the curvature factor decrease. Specifically, we capture concavity by imposing the restriction  $\lambda_{i,2}^f - \lambda_{i-1,2}^f < \lambda_{i-1,2}^f - \lambda_{i-2,2}^f$  for  $i = 3, \dots, N - 1$ .<sup>4</sup>

Employing cumulative shares in place of individual shares allows for capturing a reallocation effect across the entire spending distribution. For example, suppose all loadings on the first factor,  $f_1$ , are negative but follow an increasing trend. This configuration with individual shares would imply that all shares are uniformly declining, failing to capture any meaningful reallocation effect. In contrast, if cumulative shares have loadings that are negative but increasing, it signals that the first share has declined, while subsequent shares have increased, reflecting a redistribution from the first component to other components. Moreover, using cumulative shares enables the model to impose intuitive restrictions, such as increasing or decreasing differences, to capture curvature. In contrast, individual shares, even in an ordered sequence, make it challenging to capture convexity or concavity, as they lack the cumulative structure needed to highlight these differences.

### 4.3 Bayesian Estimation

We employ a likelihood-based Gibbs sampling approach to estimate the FAVAR model, following the methodology outlined in Bernanke et al. (2005). This iterative method provides an empirical approximation of the marginal posterior densities of both factors and parameters. As explained in Appendix B, the process involves sampling the factors based on the latest parameter draws, followed by sampling the parameters conditional on the most recent factor estimates.

The factors are drawn using the standard Kalman filter, consistent with the literature (Kim and Nelson, 1999; Bernanke et al., 2005). After drawing the factors, we proceed to sample the parameters of the measurement equation. Specifically, we draw the differences between successive loadings for each factor. The process of sampling the error variance-

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<sup>4</sup>If the shares were instead arranged in ascending order, the differences between the loadings would need to increase to reflect convexity.

covariance matrix  $R$  follows standard procedures outlined in the literature. For  $f_1$ , the differences between loadings are drawn to be greater than zero, ensuring that the loadings on  $f_1$  increase as the cumulative shares grow monotonically. For  $f_2$ , the differences between the loadings are drawn such that they exhibit progressively smaller differences, capturing the concavity of descending shares.

For the transition equation, we follow the methodology detailed in Chan and Jeliaskov (2009) to sample  $Q$ , adhering to the necessary restrictions for shock identification. Conditional on  $Q$ , the factors and the data, we then sample VAR coefficient matrix  $\Phi$ . This process constitutes a single iteration, which we repeat 30,000 times, discarding the initial 15,000 samples.

## 4.4 Impulse Responses to Fiscal Shocks

### Impulse Responses of Aggregates in $Y$

We turn our attention to estimating the dynamic responses of the variables in  $Z_t = \begin{bmatrix} f_t' & Y_t' \end{bmatrix}'$ , which comprises the first  $(K + M)$  equations of the VAR system, to surprise shifts in the reallocation factor ( $f_1$ ) and the level of spending ( $G$ ). This requires making identifying assumptions that allow us to extract the innovations embedded in the reduced-form errors. Following Chan and Jeliaskov (2009), the  $(K + M) \times (K + M)$  variance-covariance matrix ( $Q$ ) of the reduced-form errors in equation (2) can be decomposed as:

$$\begin{aligned} Q &= L^{-1}DL^{-1'} \\ v_t &= L^{-1}D^{1/2}\varepsilon_t \\ \varepsilon_t &\sim N(0, I) \end{aligned} \tag{3}$$

$\varepsilon_t$  is a  $(K + M) \times 1$  vector of structural shocks that are mutually independent.  $D$  is a diagonal matrix with positive diagonal elements, and  $L$  is a lower triangular matrix with ones on the main diagonal. Furthermore, we set the off-diagonal elements of  $L$  to zero for  $f_1$ ,  $f_2$  and  $G$ . These restrictions imply that a shock to  $G$  represents a pure level shock, as it does not contemporaneously affect the spending proportions or the relative importance of components. Conversely, a shock to  $f_1$  can be interpreted as a pure reallocation shock, affecting the spending proportions across components without impacting total spending or

the overall shape of the distribution.<sup>5</sup> This approach is particularly advantageous for analyzing exogenous shifts within distinct categories of government spending, without requiring an increase in the overall spending level.

For every  $(d + 1)^{th}$  iteration, we estimate the corresponding diagonal and lower triangular matrices,  $D^{(d+1)}$  and  $L^{(d+1)}$ , respectively, which then yields  $Q^{(d+1)}$ . Using the estimates of  $Q^{(d+1)}$  and  $\Phi^{(d+1)}$ , we derive the impulse responses to both the level shock and the reallocation shock for each iteration, facilitating a comprehensive analysis of their effects on macroeconomic aggregates.

### Impulse Responses of Cumulative Shares

After determining the impulse responses for the factors ( $f_t$ ) and observable aggregates ( $Y_t$ ), we calculate the impulse responses for the cumulative shares using the measurement equation (1), which linearly links the cumulative shares to the latent factors. For each  $(d + 1)^{th}$  iteration, we use the estimated factor loadings  $\Lambda^{f^{(d+1)}}$  and corresponding responses of the two factors.

### Impulse Responses of Individual Shares

The impulse responses of individual shares can be derived from the responses of the cumulative shares. For the first component, the impulse response is directly obtained from the first cumulative share since  $g_1 = s_1$ . For the remaining components, we calculate the impulse responses by subtracting the responses of consecutive cumulative shares, as  $s_{i,t} = g_{i+1,t} - g_{i,t}$  for  $i = 2, \dots, N - 1$ . Given the imposed restrictions, a positive reallocation shock will result in a decrease in the share  $s_1$ . Since the loadings on the reallocation factors are increasing, the cumulative shares will also increase monotonically. This implies that the individual shares  $s_i$  for  $i = 2, \dots, N - 1$  will rise, indicating a shift in spending distribution from  $G_1$  to other components. Finally, because the sum of all individual shares must equal 1, the impulse response of the  $N^{th}$  component's share is obtained by taking the negative of the responses of the cumulative share  $\left(g_{N-1} = \sum_{i=1}^{N-1} s_i\right)$ .<sup>6</sup> We derive the impulse responses for the individual shares of each of the  $N$  components in every iteration.

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<sup>5</sup>Similarly, a shock to the curvature factor ( $f_2$ ) alters the relative importance of intermediate shares while leaving the average share and total spending unchanged.

<sup>6</sup>The sign of the responses of  $s_N$  cannot be determined as it would depend on the sign of the responses of  $g_N - 1$ .

## Impulse Responses of Individual Components

Having ascertained the impulse responses for both individual shares and aggregate spending, we compute the impulse responses for spending within each components using the equation:

$$G_{i,t} = s_{i,t} \times G_t$$

When considering all three variables in natural logs, the impulse response of spending in each component is the sum of the impulse responses of its share and total spending. However, because the responses of individual shares represent percentage point changes, we normalize these by dividing by the average share over the sample period to convert them into percentage changes. As before, we compute the impulse responses for each component ( $G_i$ ) in every iteration.

Lastly, we display the median impulse responses of all variables along with their respective 68% credible sets.

## Multipliers

While impulse responses offer insight into the dynamic effects of reallocation, we also employ multipliers to quantify these effects more broadly, providing a complementary perspective on the overall impact. A fiscal multiplier measures the change in output for a unit increase in fiscal spending. The multiplier can be assessed by examining the output response at a specific point in time following an initial movement in government spending, which is also referred to as the impact multiplier (Blanchard and Perotti, 2002; Mountford and Uhlig, 2009; Ilzetzki et al., 2013). This approach does not account for any negative output responses that may develop over time. To address this limitation, the multiplier can alternatively be calculated by considering the output responses over an extended horizon, thereby reflecting how output adjusts to the evolving changes in government spending from the initial impact to the end of the horizon (Auerbach and Gorodnichenko, 2012; Laumer and Philipps, 2020). For comparison with the results of Ellahie and Ricco (2017), who provide multipliers for disaggregated fiscal components, we adopt their method.

**Pure Level Multiplier** - We calculate the multiplier in response to a shock to the level factor ( $G$ ) by considering the net present value of the cumulative change in  $GDP$  per unit

of additional government expenditure  $G$ , also discounted, over a horizon  $K$ , expressed as:<sup>7</sup>

$$Pure\ Level\ Multiplier(K) = \frac{\sum_{\kappa=0}^K (1 + \bar{r})^{-\kappa} y_{\kappa}}{\sum_{\kappa=0}^K (1 + \bar{r})^{-\kappa} G_{\kappa}} \times \frac{\bar{y}}{\bar{G}} \quad (4)$$

Here,  $\bar{r}$  denotes the average federal funds rate over the sample.  $y_{\kappa}$  and  $G_{\kappa}$  are the output and the aggregate government spending responses for a particular horizon  $\kappa$ .  $\frac{\bar{y}}{\bar{G}}$  is the sample mean of the GDP to government spending ratio. Since we consider these variables in natural logarithms, the impulse responses represent elasticities and are thus scaled by the sample mean ratio.

**Pure Reallocation Multiplier** - For calculating the present value multiplier resulting from a reallocation shock, specifically a shock to  $f_1$ , we obtain the impulse responses of  $G_1$  as detailed in the earlier section. We consider the first component because an unexpected increase in the reallocation factor leads to a decrease in the share of the  $G_1$ . Meanwhile, the shares of remaining  $N - 1$  components increase in response to the reallocation shock, indicating a redistribution from  $G_1$  to the others. Using  $G_{1,\kappa}$  to denote the response of component  $G_1$  over  $\kappa$  to the reallocation shock, the multiplier can be computed as follows:

$$Pure\ Reallocation\ Multiplier(K) = \frac{\sum_{\kappa=0}^K y_{\kappa}}{\sum_{\kappa=0}^K G_{1,\kappa}} \times \frac{\bar{y}}{\bar{G}_1} \quad (5)$$

In this equation,  $\frac{\bar{y}}{\bar{G}_1}$  represents the sample mean of the GDP to the fiscal spending component,  $G_1$ .

We compute the level and reallocation multipliers for every  $(d + 1)^{th}$  iteration using the impulse responses corresponding to that iteration. We report the median multiplier for different values of  $K$  along with the respective 68% credible sets in all cases.<sup>8,9</sup>

## 5 Results

This section presents our findings. We compare the results of the level shock with existing literature to assess how the inclusion of reallocation between fiscal components impacts the

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<sup>7</sup>The present value multipliers were introduced by Mountford and Uhlig (2009)

<sup>8</sup>Additionally, we report multipliers with no discounting ( $\bar{r} = 0$ ) as well.

<sup>9</sup>Given that the multiplier is influenced by the sample mean of the output-to-spending ratio, which can vary across different sample periods, we continue to use this formula to facilitate comparison with previous studies that employ the same approach.

findings. We then analyze the dynamic responses to reallocation shocks. Although there are no empirical counterparts to our reallocation exercise, this analysis can provide valuable insights for important budgetary decisions made by fiscal authorities.

## 5.1 Responses to a Level Shock

### Responses of Aggregates

Figure 2.1 illustrates the median impulse responses following a shock to total government spending ( $G$ ), accompanied by 16%–84% credible bands. Initially, government spending increases by 0.8%, peaking at a 1% increase after four quarters, after which it gradually declines, remaining statistically significant throughout the response period. Additionally, due to our restrictions, the level shock does not contemporaneously affect the reallocation (and curvature) factor. However, we observe that neither factor responds significantly at any point during the response horizon.

The fiscal deficit increases immediately in response to a positive shock to government spending, reaching a peak increase of 0.3 percentage points. This pattern closely follows the trajectory of  $G$ , indicating that short-term spending increases are financed through higher government debt. Output initially rises by 0.2%, remains around 0.2% for four quarters, and then begins to decline, with this significant increase in output dissipating after four quarters.

The output multipliers in response to this level shock are positive but diminish over time as the increase in output declines, as shown in column (1) of Table 1. Over a five-year horizon, the discounted multiplier is 0.42. Although this shock to  $G$  is not directly comparable to standard government spending shocks in the literature, our result is close to the 0.35 multiplier reported by Ellahie and Ricco (2017). Similarly, our non-discounted five-year multiplier of 0.56, presented in column (1) of Table 2, aligns with the 0.57 multiplier estimated by Auerbach and Gorodnichenko (2012). In contrast, Ramey (2011) finds that the government spending multiplier ranges from 0.6 to 1.2, depending on the sample analyzed.

### Responses of Disaggregated Components

Figure 2.3 displays the median impulse responses of the six component shares to a shock in  $G$ , along with the 16%–84% credible intervals.<sup>10</sup> Since the level shock alters the total spending level while preserving the contemporaneous proportions of spending, the individual shares

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<sup>10</sup>The response of  $s_6$  reflects the inverse of  $g_5$ 's response, as the shares collectively sum to 1.

do not react on impact. Furthermore, these shares exhibit no significant response over time, as reflected in the responses of the cumulative shares depicted in Figure 2.2. Consequently, the extracted factors, as illustrated in Figure 2.1, also remain unaffected by the level shock.

As shown in Figure 2.4, the increase in total government spending leads to higher spending across individual components. Each component increases significantly by 0.8% on impact, reflecting  $G$ 's response to its own shock. The responses of the components remain positive throughout the entire forecast horizon, but only four components experience statistically significant increases. Defense consumption and state and local government investment peak at approximately 2% after three quarters before converging to pre-shock levels, with the increase remaining significant from the third to the tenth quarter. Federal non-defense consumption and defense investment increase by 1.2% and 1.4%, respectively, before gradually returning toward zero, with significance lasting sixteen quarters for the former and fourteen for the latter.

## 5.2 Responses to a Reallocation Shock

### Responses of Agregates

Under the limitations placed on the loadings for the initial share ( $\lambda_{1,1}^f = -1$ ;  $\lambda_{1,2}^f = 0$ ), a shock to the reallocation factor requires a decrease in the proportion of spending assigned to the first component.

In Figure 3.1, a disturbance to the reallocation factor instantly boosts the reallocation factor by 10 percentage points, followed by a steep drop to 2.6 percentage points after one quarter. The value of  $f_1$  then increases again, reaching a peak of 4 percentage points, before gradually returning toward zero. This impact remains notable for as long as 18 quarters. Since the disturbance causes only a minimal change in the curvature factor, the rising  $f_1$  loadings for the following cumulative shares increase the shares of other components ( $s_i$ ), signifying a shift in spending away from the first component to the other components.

Government expenditure, which does not react immediately, starts to increase, peaking at 0.6% before slowly declining, with this rise remaining statistically significant throughout. The fiscal deficit drops by 0.5 percentage points immediately due to a reallocation of spending from state and local government consumption to other areas, including government investment. As investment spending occurs over longer periods, this transition contributes to a short-term decrease in the fiscal deficit. As  $G$  increases, the deficit also rises, but this



reaction is statistically significant only in the first quarter. Meanwhile, the GDP response is positive throughout the forecast horizon, although it is only significant in the first quarter. The initial significant response sees GDP increasing by 0.2%, despite no immediate alteration in total spending, arising from reduced expenditures on state and local government consumption and heightened spending on other components.

The reallocation multiplier is negative and shows a downward trend over time, as illustrated in column (2) of Table 1. It is statistically significant only at the outset, with a value of -0.40 over 20 quarters. This negative multiplier suggests that reducing the share of state and local government consumption while increasing the share of other components could be advantageous. With  $N > 2$  components, the output response is contingent upon which specific component shares increase or decrease, and the magnitudes of said changes, in reaction to a reduction in the first component's share.

### Responses of Disaggregated Components

Figure 3.2 displays the median impulse responses of the five cumulative shares to a shock in  $f_1$ , along with 16%–84% credible intervals. The response of the first cumulative share,  $g_1$ , reflects the inverse of the reallocation factor's response to its shock. As the loadings on  $f_1$  progressively increase across the cumulative shares, each subsequent share experiences a smaller initial decline but follows a similar trajectory as  $g_1$ . Specifically,  $g_2$  decreases by 4.7 percentage points,  $g_3$  by approximately 2.7 points,  $g_4$  by about 2.4 points, and  $g_5$  by 1.7 percentage points. Additionally, as we move from  $g_2$  to  $g_5$ , the duration of the significant decline shortens: it lasts for approximately 16 quarters in  $g_2$ , around 12 quarters in  $g_3$ , 10 quarters in  $g_4$ , and nearly 5 quarters in  $g_5$ .

The pattern of progressively smaller declines in the cumulative shares suggests an increase in component shares upon impact. As illustrated in Figure 3.3, all component shares except  $s_1$  increase following the reallocation shock, which aligns with expectations. The federal defense consumption share rises by 5.3 percentage points upon impact, then declines to 1.3 percentage points, before increasing over three quarters and gradually returning to zero. This rise remains statistically significant for 16 quarters. Other shares exhibit similar response patterns:  $s_3$  increases by 2 percentage points upon impact,  $s_4$  by 0.3 percentage points, and  $s_5$  by 0.7 percentage points. The statistical significance of these increases varies:  $s_3$ 's rise remains significant for nearly 16 quarters,  $s_4$ 's for 2 quarters, and  $s_5$ 's for approximately 6 quarters. Federal non-defense investment, mirroring the inverse response of  $g_5$ , rises whenever

$s_6$  declines, ensuring that the component shares sum to 1. Consequently,  $s_6$  increases by 1.7 percentage points upon impact and returns to baseline after six to seven quarters.

The responses of each component are directly aligned with changes in their respective shares. The reduction in spending for state and local government consumption is offset by increases in other components, resulting in a significant overall rise in government spending in response to the reallocation shock.

$G_1$  initially declines by 23% upon impact, following a pattern consistent with its share. The sharp initial decline partially reverses and gradually returns to zero after 12 quarters, indicating a possible reduction in federal support for state and local governments. On impact, defense consumption rises significantly by 23%, state and local government investment by 17%, non-defense consumption by 3%, and defense investment by 10%. These increases gradually decline and converge to zero after roughly 12 to 18 quarters. Additionally, non-defense investment by the federal government jumps by 39% upon impact, returning to baseline after 10 quarters.

The shift in spending distribution away from state and local government consumption contributed to a 0.2% increase in GDP and a 0.5 percentage point decline in the fiscal deficit.

In summary, decreasing the share of state and local government consumption results in increased shares across all other components. This reallocation significantly enhances overall spending without a corresponding rise in the fiscal deficit, thereby boosting output. These findings indicate that a cross-sectional shift in spending distribution can positively affect output, with the outcomes influenced by both the direction and magnitude of the shift.

### 5.3 Robustness Check: Reordering the Component Shares

In Appendix C, we arrange component shares in ascending order, defining them as follows: (1)  $s_1 = NDEFI/G$ , (2)  $s_2 = DEFI/G$ , (3)  $s_3 = NDEFC/G$ , (4)  $s_4 = SLGI/G$ , (5)  $s_5 = DEFC/G$ , and (6)  $s_6 = NDEFC/G$ . Consequently, the loadings on  $f_2$  in equation (1) are constructed so that the differences between consecutive loadings increase, reflecting the convexity of the distribution.

A positive level shock elicits responses for  $G$ , the fiscal deficit, and GDP that are quantitatively similar to previous findings, with a discounted level multiplier of 0.47 over 20 quarters (Table C1). Regarding the reallocation shock, the decrease in  $s_1$  briefly increases the share

of spending on other components, lasting for approximately one quarter. The GDP response to the shock remains negative throughout the forecast horizon, with statistical significance observed only initially. The initial 0.05% decline in GDP, despite no immediate change in  $G$ , indicates shifts in spending across components.

When shares are ordered in descending order, a reallocation shock significantly affects the reallocation factor; in ascending order, it primarily impacts the curvature factor. This variation occurs because larger shares react more strongly to the reallocation shock, influencing the reallocation factor in descending order and the curvature factor in ascending order.

The results are influenced by which component experiences the reallocation shock, the components that see increases or decreases in response, and the magnitude of these shifts.

## 5.4 Economic Implications

In the very near term, a shift in spending from state and local government consumption (SLGC) to non-defense investment by both the federal government (NDEFI) and state and local governments (SLGI) can lead to immediate productivity-enhancing effects. Non-defense investment components tend to have higher multipliers compared to non-defense consumption expenditures by state and local governments (Ellahie and Ricco, 2017). However, the observed increase in federal defense consumption (DEFI), which generally has a lower multiplier than SLGC, underscores the importance of understanding how resources are allocated. Consequently, the effectiveness of these spending shifts largely depends on the specific components that receive increased funding.

In the short to medium term, heightened spending in other components can offset the decrease in SLGC without significantly adding to fiscal debt. GDP does not exhibit substantial growth, indicating that the scale of fiscal spending is limited. As spending in these components returns to pre-shock levels, government spending ( $G$ ) begins to decline. However, a more persistent shift in the composition of spending could generate long-term effects. Ongoing prioritization of defense spending in response to prolonged international conflicts may further constrain non-defense expenditures. This persistent trend could reduce investments in infrastructure, education, and other growth-enhancing sectors, ultimately limiting the economy's long-term productive capacity.

## 6 Conclusion

Effective fiscal policy design involves strategically allocating expenditures across various sectors, including education, healthcare, infrastructure, and defense, often while adhering to budget constraints. These constraints necessitate trade-offs among spending categories, where an increase in funding for one area may lead to reduced allocations for others. If output multipliers were the same across categories, changes in spending distribution would not affect the overall aggregate multiplier. However, the varying macroeconomic impacts of individual spending components result in different multipliers across sectors.

Our study examines how redistributing spending from one component to others influences the overall fiscal structure, acknowledging that a decrease in the proportion allocated to one category does not always imply a cut in spending. It may indicate that spending in that category is growing at a slower rate than total government spending. Furthermore, the proportions of different components can shift at different rates, with the pace of these changes also varying. Our innovative framework captures both the shifts in component shares and the differing rates of change by employing the slope (reallocation) factor and the curvature factor. It also reflects spending adjustments across components by considering changes in component shares alongside the overall spending level.

By ranking the six main components of government spending from largest to smallest, we analyze the effects of reallocating funds from the largest component—state and local government consumption—to the other five. This reallocation increases the shares of these five components, raising total spending without adding to the fiscal deficit. It also leads to a significant rise in output, indicating that reallocating funds away from state and local government consumption may be advantageous. In contrast, reallocating from the smallest component, federal non-defense investment, results in a considerable increase in the fiscal deficit and a decrease in output, suggesting that reducing non-defense investment may not be beneficial. This approach can be adapted to assess any number or type of disaggregated fiscal components.

Our findings indicate that considering the distribution and interaction of spending components can improve the effectiveness of fiscal policy. Taking into account time variation in spending composition may help reconcile differences in multiplier estimates across periods. Future research should investigate the use of a more flexible model that incorporates multiple spending components with fewer constraints.

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# Figures and Tables

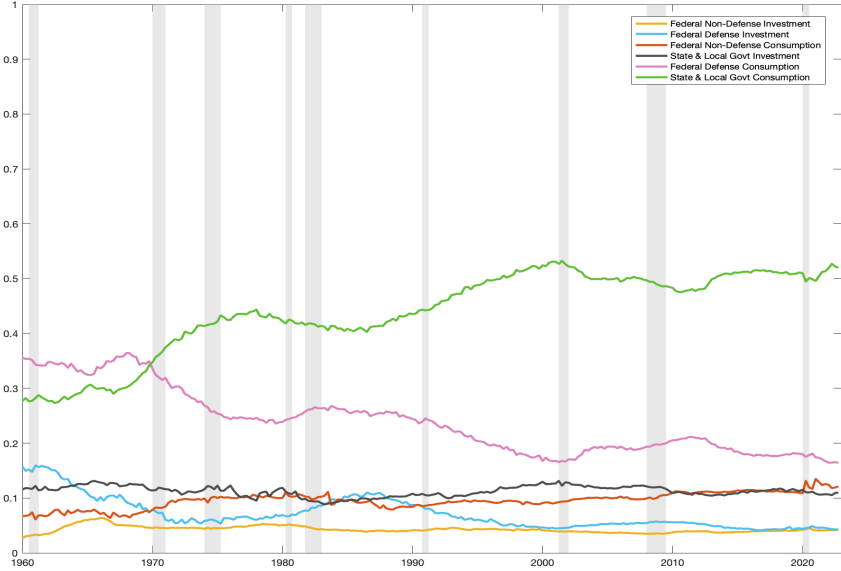


Figure 1: Components of Government Spending Fraction of Total Government Spending

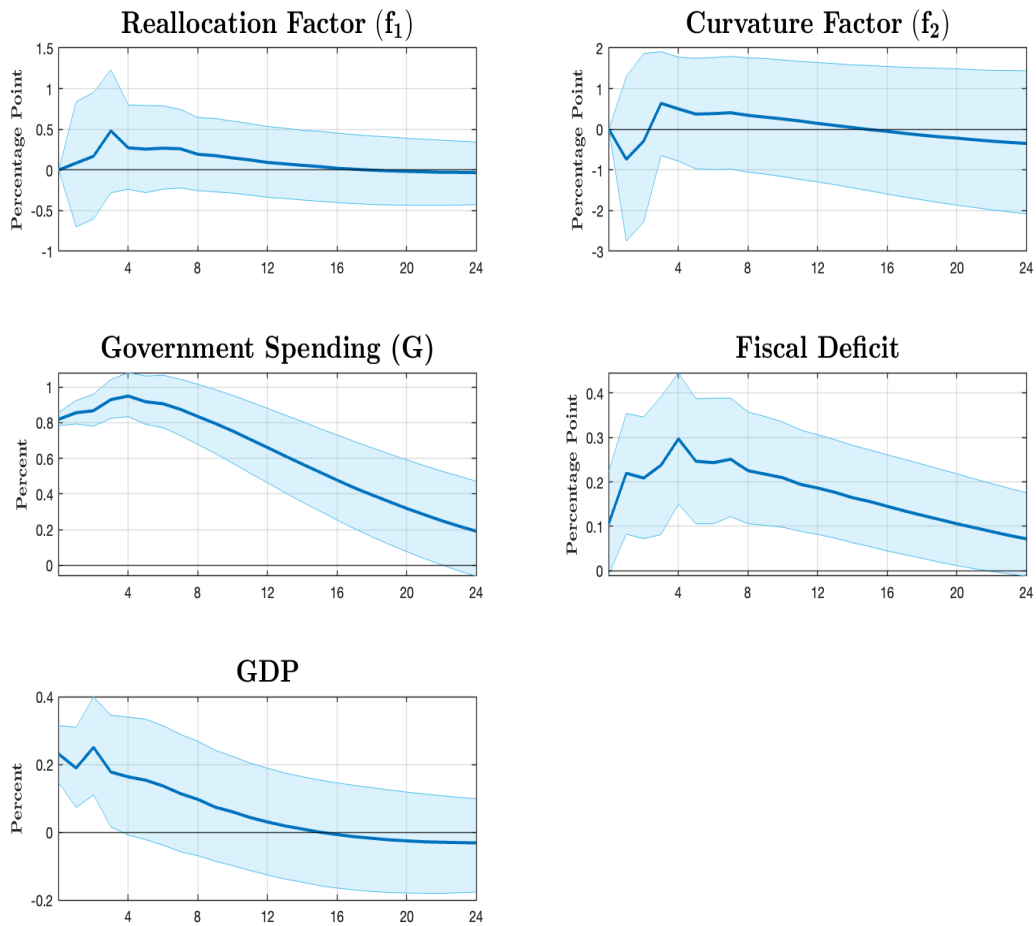


Figure 2.1: Impulse responses to a level shock (shock to  $G$ ) generated from FAVAR with two factors described by equations (2) and (3). Here, the level ( $G$ ) represents total government spending, which is the sum of all the six components considered. The individual shares are arranged in a descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.



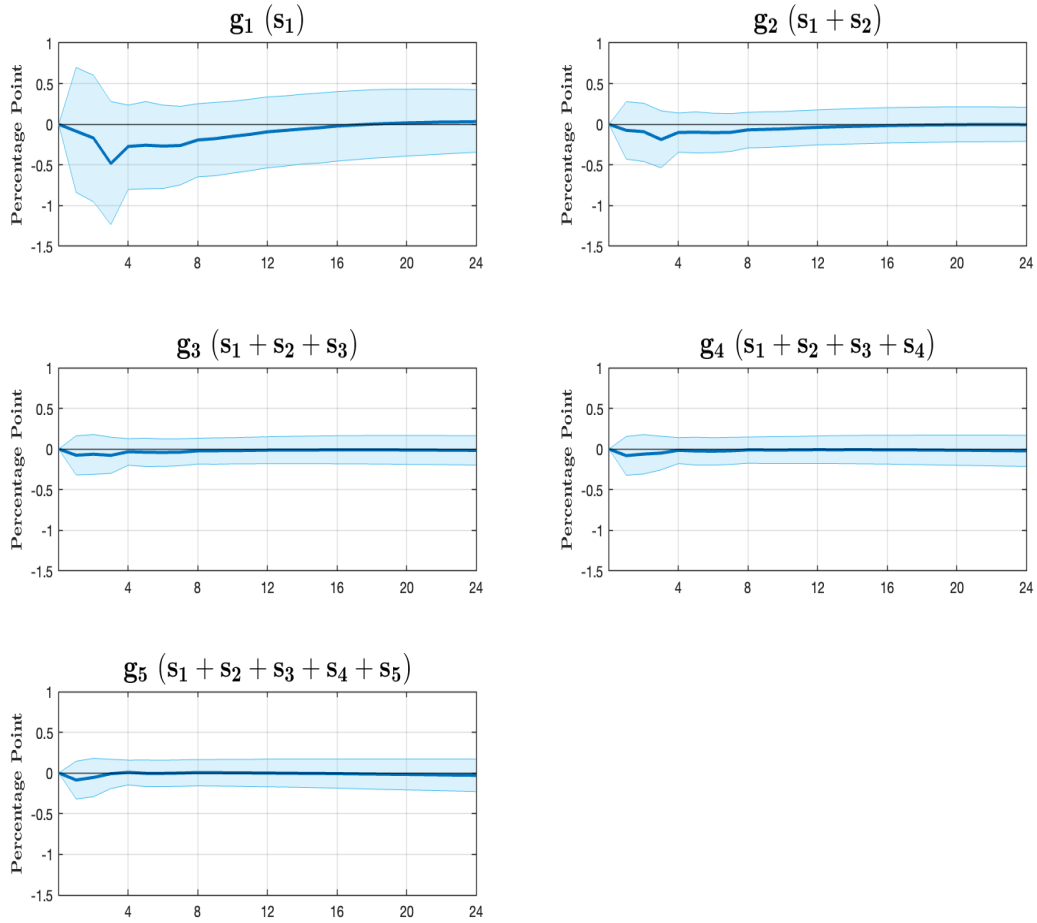


Figure 2.2: Impulse responses of cumulative shares ( $g_i$ ) to a level shock (shock to  $G$ ) generated from FAVAR with two factors described by equations (2) and (3). Here, the six individual shares arranged in ascending order are defined as follows - (1)  $s_1 = SLGC/G$ , (2)  $s_2 = DEFC/G$ , (3)  $s_3 = SLGI/G$ , (4)  $s_4 = NDEFC/G$ , (5)  $s_5 = DEFI/G$  and (6)  $s_6 = NDEFI/G$ . Here,  $G$  represents total government spending,  $SLGC$  is State and Local Government Consumption,  $DEFC$  is Federal Defense Consumption,  $SLGI$  is State and Local Government Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $DEFI$  is Federal Defense Investment and  $NDEFI$  is Federal Non-Defense Investment. The individual shares are arranged in descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

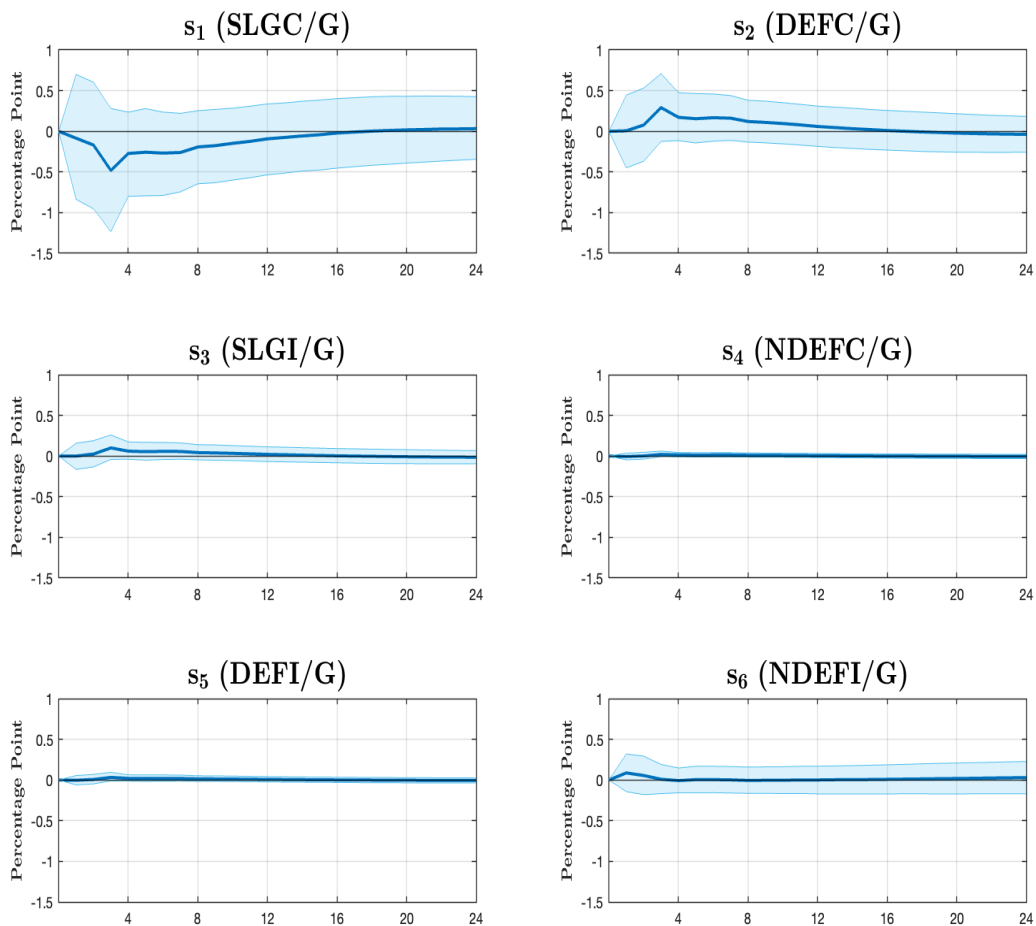


Figure 2.3: Impulse responses of individual shares ( $s_i$ ) to a level shock (shock to  $G$ ) generated from FAVAR with two factors described by equations (2) and (3). Here,  $G$  represents total government spending,  $SLGC$  is State and Local Government Consumption,  $DEFC$  is Federal Defense Consumption,  $SLGI$  is State and Local Government Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $DEFI$  is Federal Defense Investment and  $NDEFI$  is Federal Non-Defense Investment. The individual shares are arranged in descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

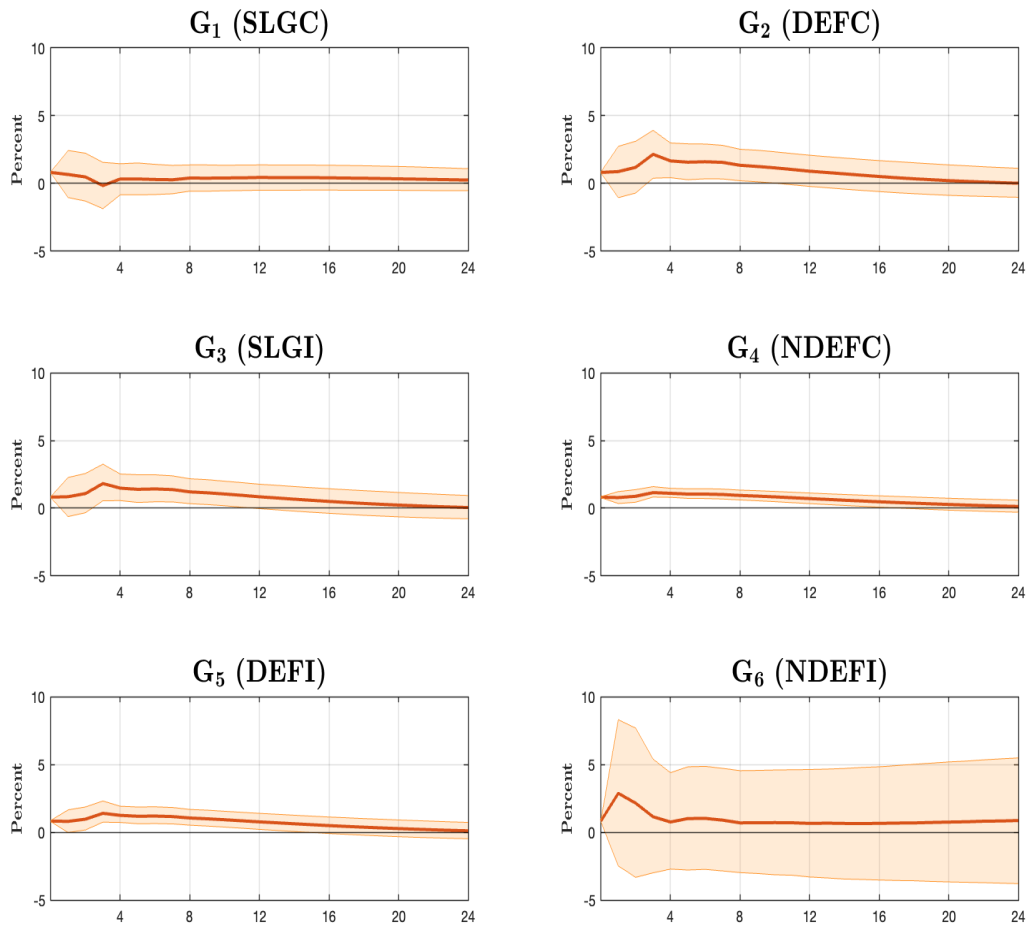


Figure 2.4: Impulse responses of spending on components ( $G_i$ ) to a level shock (shock to  $G$ ) generated from FAVAR with two factors described by equations (2) and (3). Here,  $G$  represents total government spending,  $SLGC$  is State and Local Government Consumption,  $DEFC$  is Federal Defense Consumption,  $SLGI$  is State and Local Government Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $DEFI$  is Federal Defense Investment and  $NDEFI$  is Federal Non-Defense Investment. The individual shares are arranged in descending order. Orange solid lines represent medians, and the shaded area represents the 68% credible bands.

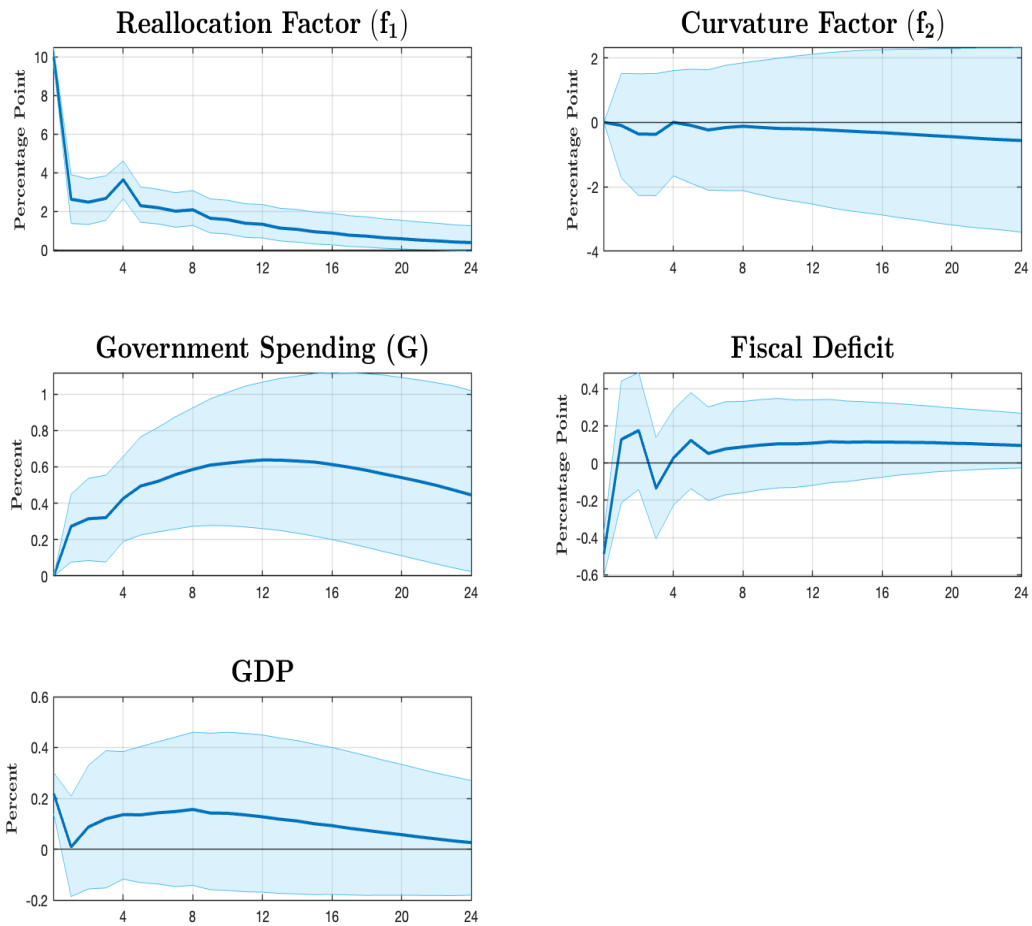


Figure 3.1: Impulse responses to a reallocation shock (shock to  $f_1$ ) generated from FAVAR with two factors described by equations (2) and (3). Here, the level ( $G$ ) represents total government spending, which is the sum of all the six components considered. The individual shares are arranged in a descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

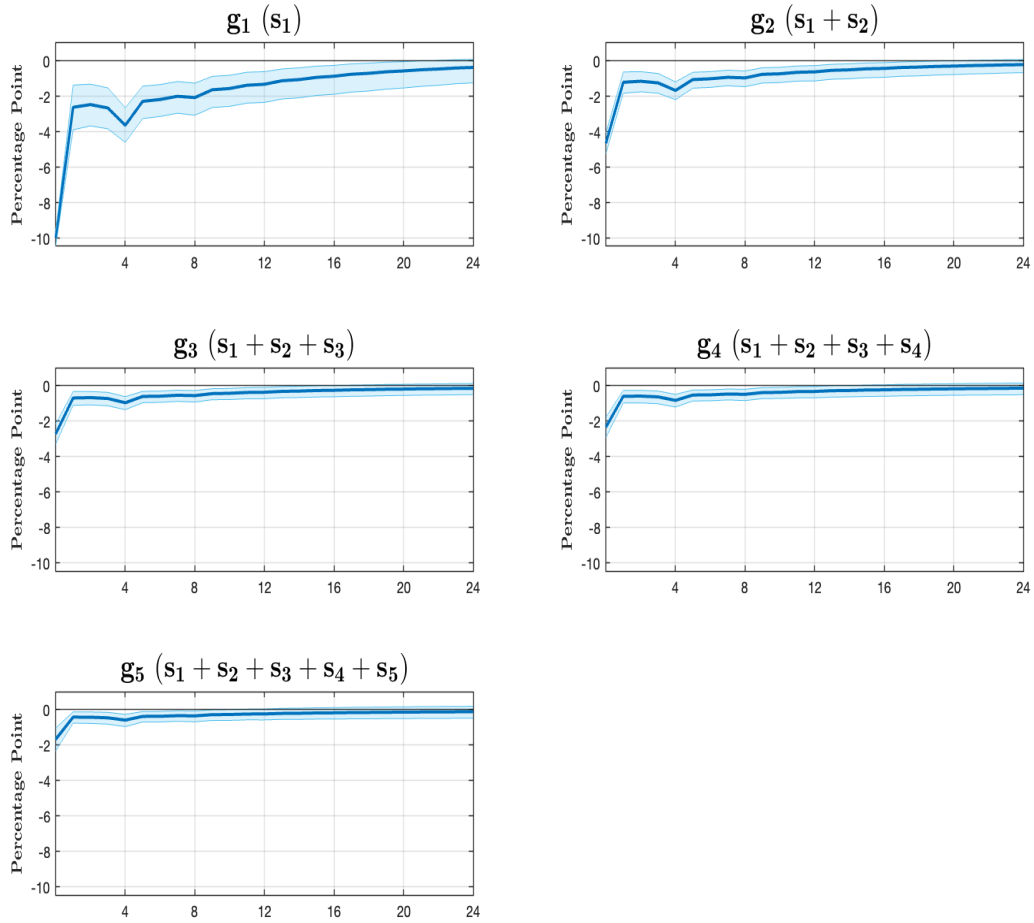


Figure 3.2: Impulse responses of cumulative shares ( $g_i$ ) to a reallocation shock (shock to  $f_1$ ) generated from FAVAR with two factors described by equations (2) and (3). Here, the six individual shares arranged in ascending order are defined as follows - (1)  $s_1 = SLGC/G$ , (2)  $s_2 = DEFC/G$ , (3)  $s_3 = SLGI/G$ , (4)  $s_4 = NDEFC/G$ , (5)  $s_5 = DEFI/G$  and (6)  $s_6 = NDEFI/G$ . Here,  $G$  represents total government spending,  $SLGC$  is State and Local Government Consumption,  $DEFC$  is Federal Defense Consumption,  $SLGI$  is State and Local Government Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $DEFI$  is Federal Defense Investment and  $NDEFI$  is Federal Non-Defense Investment. The individual shares are arranged in descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

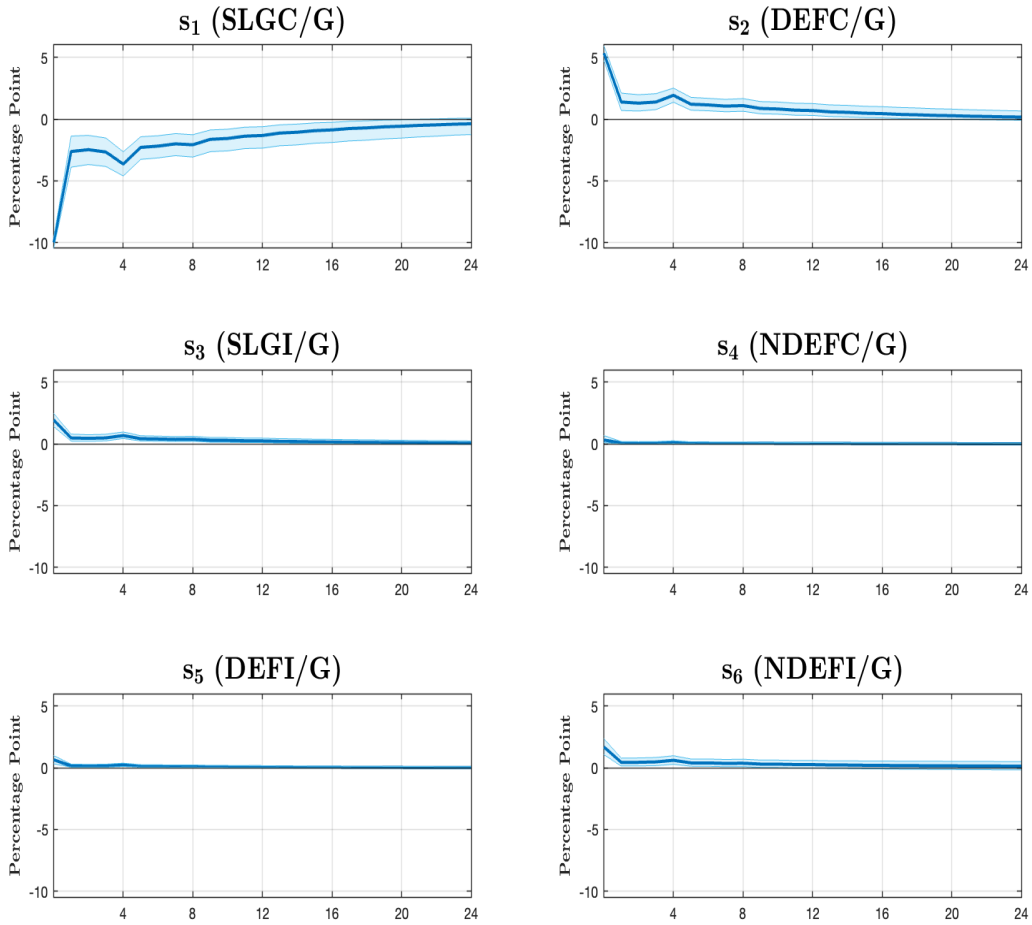


Figure 3.3: Impulse responses of individual shares ( $s_i$ ) to a reallocation shock (shock to  $f_1$ ) generated from FAVAR with two factors described by equations (2) and (3). Here,  $G$  represents total government spending,  $SLGC$  is State and Local Government Consumption,  $DEFC$  is Federal Defense Consumption,  $SLGI$  is State and Local Government Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $DEFI$  is Federal Defense Investment and  $NDEFI$  is Federal Non-Defense Investment. The individual shares are arranged in descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

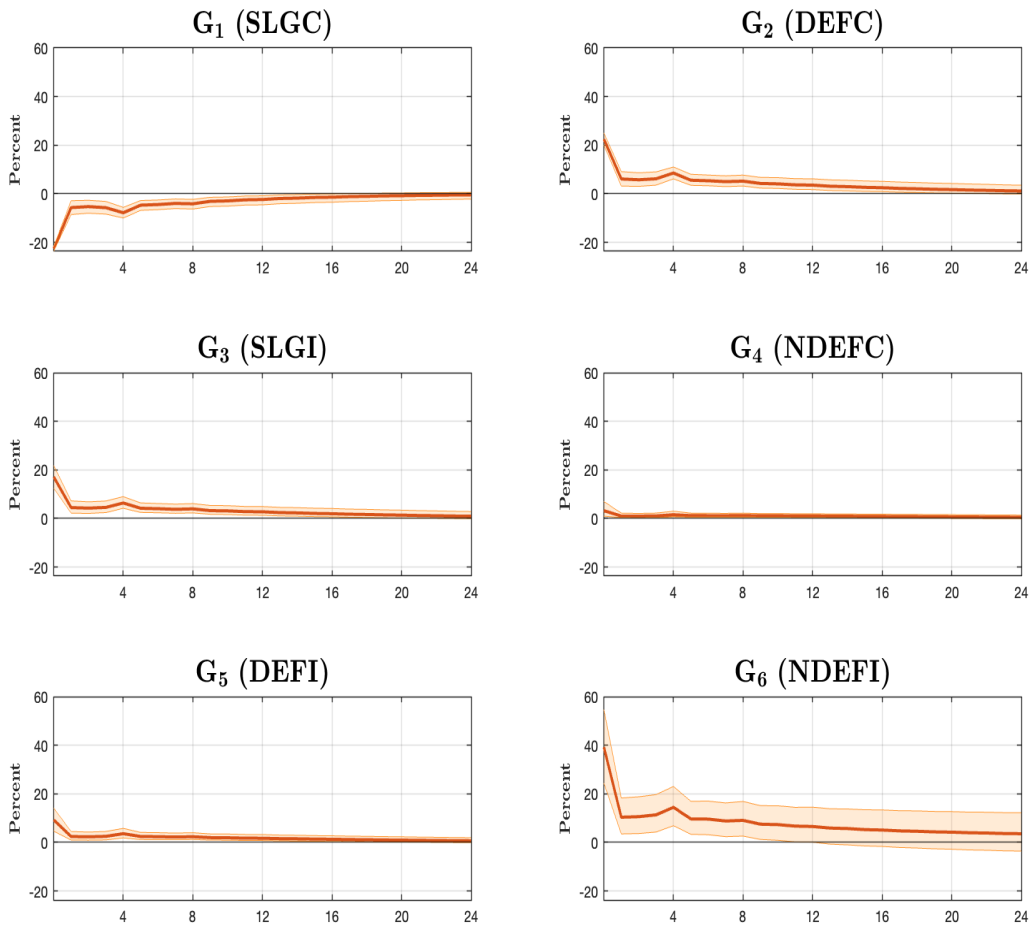


Figure 3.4: Impulse responses of spending on components ( $G_i$ ) to a reallocation shock (shock to  $f_1$ ) generated from FAVAR with two factors described by equations (2) and (3). Here,  $G$  represents total government spending,  $SLGC$  is State and Local Government Consumption,  $DEFC$  is Federal Defense Consumption,  $SLGI$  is State and Local Government Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $DEFI$  is Federal Defense Investment and  $NDEFI$  is Federal Non-Defense Investment. The individual shares are arranged in descending order. Orange solid lines represent medians, and the shaded area represents the 68% credible bands.

Table 1: Discounted Cumulative Multipliers

This table presents the discounted cumulative multipliers for GDP across the full sample at different horizons with 68% credible bands in brackets. Columns (1) and (2) represent discounted multipliers in response to a level shock (shock to  $G$ ) and a reallocation shock (shock to  $f_1$ ) when the individual shares are arranged in descending order.

Horizon	Level multiplier (1)	Reallocation multiplier (2)
On Impact	1.40 [0.89, 1.91]	-0.11 [-0.15, -0.07]
1 quarter	1.24 [0.69, 1.80]	-0.09 [-0.19, 0.01]
4 quarters	1.12 [0.45, 1.81]	-0.14 [-0.38, 0.09]
8 quarters	0.92 [0.14, 1.65]	-0.22 [-0.61, 0.15]
12 quarters	0.74 [-0.12, 1.48]	-0.29 [-0.87, 0.23]
16 quarters	0.57 [-0.39, 1.34]	-0.35 [-1.15, 0.33]
20 quarters	0.42 [-0.73, 1.27]	-0.38 [-1.46, 0.44]
24 quarters	0.30 [-1.14, 1.24]	-0.39 [-1.77, 0.58]



Table 2: Non-discounted Cumulative Multipliers

This table presents the discounted cumulative multipliers for GDP across the full sample at different horizons with 68% credible bands in brackets. Columns (1) and (2) represent discounted multipliers in response to a level shock (shock to  $G$ ) and a reallocation shock (shock to  $f_1$ ) when the individual shares are arranged in descending order.

Horizon	Level multiplier (1)	Reallocation multiplier (2)
On Impact	1.40 [0.89, 1.91]	-0.11 [-0.15, -0.07]
1 quarter	1.25 [0.69, 1.80]	-0.09 [-0.19, 0.01]
4 quarters	1.14 [0.47, 1.81]	-0.14 [-0.36, 0.08]
8 quarters	0.95 [0.19, 1.66]	-0.20 [-0.56, 0.13]
12 quarters	0.80 [-0.02, 1.51]	-0.26 [-0.75, 0.19]
16 quarters	0.66 [-0.21, 1.40]	-0.30 [-0.92, 0.25]
20 quarters	0.56 [-0.41, 1.32]	-0.32 [-1.08, 0.32]
24 quarters	0.48 [-0.62, 1.28]	-0.33 [-1.22, 0.38]

# Appendix

## A Variable Definitions

1. **Government Spending Components:** Quarterly data for each of the components of government purchases are sourced from Table 3.9.5 of the National Income and Product Accounts (NIPA), provided by the Bureau of Economic Analysis. Details for each component are as follows:

- Federal Defense Consumption Expenditures (DEFC) - Row 18
- Federal Defense Gross Investment (DEFI) - Row 19
- Federal Non-Defense Consumption Expenditures (NDEFC) - Row 26
- Federal Non-Defense Gross Investment (NDEFI) - Row 27
- State and Local Government Consumption Expenditures (SLGC) - Row 34,
- State and Local Government Gross Investment (SLGI) - Row 35

All components are adjusted to real per capita terms by dividing the nominal values by the GDP deflator and the population measure. The sum of these components constitutes total government spending. To calculate total federal spending, we aggregate the federal defense and federal non-defense components. Similarly, state and local government spending is computed by summing the respective consumption and investment figures.

2. **Population:** Quarterly population data, including resident population plus armed forces overseas from the Federal Reserve Board of St Louis website (B230RC0Q173SBEA).

3. **Nominal GDP:** This metric is available from Line 1 of Table 1.1.5 in the National Income and Product Accounts (NIPA). Real per capita values are calculated by dividing nominal values by both the GDP deflator and the population figure.

4. **Implicit Price Deflator for GDP:** This is sourced from Row 1 of Table 1.1.9 of National Income and Product Accounts (NIPA).

5. **Fiscal Deficit:** Fiscal deficit is defined as difference between federal government expenditures and federal government receipts as a percentage of nominal GDP. The

data on receipts (W018RC1Q027SBEA) and expenditures (W019RCQ027SBEA) is obtained from the Federal Reserve Board of St Louis website.

## B Bayesian Estimation

In this appendix, we elaborate on the use of the Gibbs sampling procedure in estimating our FAVAR model. For simplicity, let  $X_t = [\chi_t' \ Y_t']'$ ,  $\xi_t = [Z_t' \ \dots \ Z_{t-p+1}']'$ ,  $e_t^X = [e_t' \ 0]'$  and  $v_t^\xi = [v_t' \ 0]'$ . Therefore, equations (1) and (2) in Section 4 can be written as

$$X_t = \Lambda \xi_t + e_t^X \quad (1)$$

$$\xi_t = \Phi(L)\xi_{t-1} + v_t^\xi \quad (2)$$

where  $\Lambda$  is the loadings matrix from equation (1),  $R^X = cov(e_t^X e_t^{X'})$  represents the covariance matrix  $R$  of the measurement equation, extended with zeros beyond the  $(N - 1) \times (N - 1)$  block. Similarly,  $Q^\xi$  is the variance-covariance matrix  $Q$  of the VAR(1) process augmented with zeros beyond the  $(K + M) \times (K + M)$  block.

All of the model's parameters can be treated as random variables and are collectively referred to as  $\theta$  where  $\theta = (\Lambda, R^X, vec(\Phi), Q^\xi)$  and  $vec(\Phi)$  is the column vector of the elements of the stacked  $\Phi$  of the lag operator of the transition equation. Further, let  $\tilde{X}_T = (X_1, X_2, \dots, X_T)$  denote the history of  $X$  from periods 1 to  $T$ . Similarly, define  $\tilde{\xi}_T = (\xi_1, \xi_2, \dots, \xi_T)$ . The aim is to obtain the empirical marginal posterior densities of  $\tilde{\xi}_T$  and  $\theta$ .

The multi-move Gibbs sampler begins by selecting a set of initial values  $\theta^0$ . Second, conditional on  $\theta^0$  and data  $\tilde{X}_T$ , a set of values for  $\tilde{\xi}_T$  say  $\tilde{\xi}_T^1$ , is drawn from the conditional density  $p(\tilde{\xi}_T | \tilde{X}_T, \theta^0)$ . Then, conditional on the sampled value of the state vector  $\tilde{\xi}_T^1$  and the data, a set of values of the parameters  $\theta$  i.e.  $\theta^1$ , is drawn from the conditional distribution  $p(\theta | \tilde{X}_T, \tilde{\xi}_T^1)$ . These two steps constitute one iteration, and a total of 30,000 such iterations are performed, with the first 15,000 discarded as burn-in.

### Initial values $\theta^0$

In place of the factors, we use principal components corresponding to the largest  $K$  eigenvalues, as principal components provide a good starting value for the factors. The identification of factors using principal components is standard. Following Stock and Watson (2002), we restrict the factor loadings by setting  $\Lambda' \Lambda / N = I_K$ . This normalization identifies the factors up to a change of sign.

We impose the normalization restrictions for  $g_1$  as outlined in the previous section where we set  $\lambda_{1,1}^f = -1$  and  $\lambda_{1,2}^f = 0$ . Using the principal components in place of  $f_t$ , we get the OLS estimates of  $\Lambda$  and  $R$  with these restrictions in place. Similarly, we use the OLS estimates for  $\Phi(L)$  and  $Q$ .

## Drawing the latent factors

We follow the exact methodology described in Kim and Nelson (1999, Chapter 8) in order to sample from  $p(\tilde{\xi}_T|\tilde{X}_T, \theta)$ , assuming that the data and the hyperparameters of the model are given. By leveraging the Markov property in state space models,<sup>11</sup> the conditional distribution from which the state vector is generated can be expressed as a product of conditional distributions (equation 8.8 in Kim and Nelson, 1999), as follows.

$$p(\tilde{\xi}_T|\tilde{X}_T, \theta) = p(\xi_T|\tilde{X}_T, \theta) \prod_{t=1}^{T-1} p(\xi_t|\xi_{t+1}\tilde{X}_T, \theta) \quad (3)$$

The state space model is linear and Gaussian, hence we have:

$$\xi_T|\tilde{X}_T, \theta \sim N(\xi_{T|T}, P_{T|T}) \quad (4)$$

$$\xi_t|\xi_{t+1}, \tilde{X}_t, \theta \sim N(\xi_{t|\xi_{t+1}}, P_{t|\xi_{t+1}}) \quad \text{for } t = T-1, \dots, 1 \quad (5)$$

where

$$\begin{aligned} \xi_{T|T} &= E(F_T|\tilde{X}_T, \theta) \\ P_{T|T} &= Cov(F_T|\tilde{X}_T, \theta) \\ \xi_{t|\xi_{t+1}} &= E(\xi_t|\xi_{t+1}, \tilde{X}_t, \theta) = E(\xi_t|\xi_{t+1}, \xi_{t|t}, \theta) \\ P_{t|\xi_{t+1}} &= Cov(\xi_t|\xi_{t+1}, \tilde{X}_t, \theta) = Cov(\xi_t|\xi_{t+1}, \xi_{t|t}, \theta) \end{aligned} \quad (6)$$

Here,  $\xi_{t|t}$  and  $P_{t|t}$  refer to the expected value (mean) and uncertainty (variance) of the latent variables given the current parameters and the observed data dated  $t$ . We run the Kalman filter to generate  $\xi_{t|t}$  and  $P_{t|t}$  for  $t = 1, \dots, T$ , starting with the initial values,  $\xi_{1|0} = 0_{p(K+M) \times 1}$  and  $P_{1|0} = I_{p(K+M)}$  (Bernanke et al., 2005).

$$\begin{aligned} \xi_{t|t} &= \xi_{t|t-1} + K_t \eta_{t|t-1} \\ P_{t|t} &= P_{t|t-1} - K_t \Lambda P_{t|t-1} \end{aligned} \quad (7)$$

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<sup>11</sup> $p(\xi_t|\xi_{t+1}, \dots, \xi_T, X_T, \theta) = p(\xi_t|\xi_{t+1}, X_t, \theta)$

where  $\eta_{t|t-1} = X_t - \Lambda\xi_{t|t-1}$  is the conditional forecast error and its covariance is given by  $H_{t|t-1} = \Lambda P_{t|t-1} \Lambda' + R$ . The Kalman gain, denoted by  $K_t = P_{t|t-1} \Lambda' H_{t|t-1}^{-1}$ , determines the weight to be assigned to new information contained in the conditional forecast error. Additionally,

$$\begin{aligned}\xi_{t|t-1} &= \Phi \xi_{t-1|t-1} \\ P_{t|t-1} &= \Phi P_{t-1|t-1} \Phi' + Q\end{aligned}\tag{8}$$

The last iteration of the Kalman filter provides  $\xi_{T|T}$  and  $P_{T|T}$ , which can be used to draw  $\xi_T$  using equation (B4). Once  $\xi_T$  is obtained,  $\xi_t$ ,  $t = T - 1, \dots, 1$  can be generated by running the Kalman smoother backwards, following equation (B9). This process is akin to updating an estimate of  $\xi_t$  by combining  $\xi_{t|t}$  with the additional information provided by  $\xi_{t+1}$ . The updating equations are as follows:

$$\begin{aligned}\xi_{t|t, \xi_{t+1}} &= \xi_{t|t} + P_{t|t} \Phi^{*'} (\Phi^* P_{t|t} \Phi^{*'} + Q)^{-1} (\xi_{t+1}^* - \Phi^* \xi_{t|t}) \\ P_{t|t, \xi_{t+1}} &= P_{t|t} - P_{t|t} \Phi^{*'} (\Phi^* P_{t|t} \Phi^{*'} + Q)^{-1} \Phi^* P_{t|t}\end{aligned}\tag{9}$$

where  $Q$  refers to the upper  $(K + M) \times (K + M)$  block of the variance-covariance matrix  $Q^\xi$ . Similarly,  $\Phi^*$  and  $\xi_t^*$  refer to the first  $(K + M)$  rows of  $\Phi$  and  $\xi_t$ , respectively. This situation arises when  $Q^\xi$  is singular, which occurs when the number of lags in the transition equation exceeds 1.<sup>12</sup>

## Drawing the parameters

Conditional on the factors drawn and the data for the observable macroeconomic series, we begin by drawing the parameters related to the measurement equation (B1) and the transition equation (B2) from the conditional distribution  $p(\theta | \tilde{X}_T, \tilde{\xi}_T)$ . The choice of the priors for the parameters and the resulting posterior distributions follows from Bernanke et al. (2005). With known factors, equations (B1) and (B2) amount to standard regression equations. Starting with the measurement equation, recall that the factor loadings on the  $f_1$  and  $f_2$  for the first cumulative share are set to -1 and 0. We draw the differences  $(\Delta \Lambda_i^f)$  between the loadings such that the differences are positive for  $f_1$  and the differences are

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<sup>12</sup>In line with literature (Blanchard and Perotti, 2002), we set the number of lags equal to 4. The details and the derivations of the equations (B3) to (B9) can be found in Chapters 3 and 8 of Kim and Nelson (1999).

increasing for  $f_2$ . For  $i = 2, \dots, N - 1$ , we have,

$$\begin{aligned} g_{it} &= \Lambda_i^f f_t + e_{it} \\ g_{it} &= \left( \Lambda_{i-1}^f + \Delta \Lambda_i^f \right) f_t + e_{it} \\ g_{it} - \Lambda_{i-1}^f f_t &= \delta_i^f f_t + e_{it} \end{aligned} \tag{10}$$

Since the errors are uncorrelated ( $R_{ij} = 0, i \neq j$ ), the OLS estimates of  $\hat{\delta}_i^f$  and  $\hat{e}_i$  for  $i = 2, \dots, N - 1$  are obtained for each equation separately. Following Bernanke et al. (2005), we assume conjugate priors:

$$\begin{aligned} R_{ii} &\sim iG(R_0, t_0) \\ \delta_i^f | R_{ii} &\sim N\left(\delta_{i0}^f, R_{ii} M_0^{-1}\right) \end{aligned} \tag{11}$$

These priors conform to the following posterior distribution:

$$\begin{aligned} R_{ii} | \tilde{X}_t, \tilde{\xi}_t &\sim iG(\bar{R}_{ii}, \bar{T}) \\ \delta_i^f | \tilde{X}_t, \tilde{\xi}_t, R_{ii} &\sim N\left(\bar{\delta}_i^f, R_{ii} \bar{M}_i^{-1}\right) \end{aligned} \tag{12}$$

with

$$\begin{aligned} \bar{T} &= t_0 + T \\ \bar{R}_{ii} &= R_0 + \hat{e}_i' \hat{e}_i + (\hat{\delta}_i^f - \delta_{i0}^f) \left[ M_0^{-1} + \left( \tilde{f}_T' \tilde{f}_T \right)^{-1} \right]^{-1} (\hat{\delta}_i^f - \delta_{i0}^f) \\ \bar{\delta}_i^f &= \bar{M}_i^{-1} \left( M_0 \delta_{i0}^f + (\tilde{f}_T' \tilde{f}_T) \hat{\delta}_i^f \right) \\ \bar{M}_i &= M_0 + \tilde{f}_T' \tilde{f}_T \end{aligned} \tag{13}$$

Here,  $M_0^{-1}$  denotes variance parameter in the prior for the second factor's coefficient in the  $i^{th}$  equation. The regressor for each of the  $i^{th}$  equation is denoted by  $\tilde{f}_T$ , where  $\tilde{f}_T = (f_1, f_2, \dots, f_T)$  is the history of the factors from period 1 to period T. We use a diffuse prior specification for  $R_{ii}$ , where  $R_0 = 1; t_0 = 10^{-3}$ . Similarly, we set the priors on the coefficients of the  $i^{th}$  equation,  $\delta_{i0}^f = 0$  for  $i = 2, \dots, N - 1$  and  $M_0 = 1$ . For each  $i^{th}$  equation, we accept draws if  $\delta_{i,1}^f > 0$  and  $\delta_{i,2}^f > \delta_{i-1,2}^f$ .

Next, we draw  $vec(\Phi^*)$  and  $Q$  conditional on the draws of the factors and the data.  $\Phi^*$  refers to the first  $(K + M)$  rows of the  $\Phi$  matrix, and  $Q$  is the upper  $(K + M) \times (K + M)$  block of  $Q^\xi$ . Since  $Q$  is a positive definite, it can be decomposed into unique matrices  $L$  and  $D$  such that  $Q^{-1} = L' D^{-1} L$  (Chan and Jeliazkov, 2009).  $L$  is a lower triangular matrix

with ones on the diagonal and  $D$  is a diagonal matrix with positive diagonal elements. Let the diagonal elements in  $D$  be denoted as  $\gamma_w$ ,  $nn = 1, \dots, (K + M)$  and let  $a_{nn,mm}$ ,  $1 \leq nn < mm \leq (K + M)$  denote the elements of  $L$ .

$$D \equiv \begin{bmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \gamma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_{(K+M)} \end{bmatrix}; \quad L \equiv \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{(K+M)1} & a_{(K+M)2} & \cdots & 1 \end{bmatrix} \quad (14)$$

The priors are given as follows:

$$\begin{aligned} \gamma_{nn} &\overset{Ind}{\sim} IG\left(\frac{1+nn}{2}, \frac{q_{nn}}{2}\right), \quad nn = 1, \dots, (K + M) \\ a_{nn} &\overset{Ind}{\sim} N(0, A_0), \quad nn = 4, \dots, (K + M) \\ \text{vec}(\Phi^*)|Q &\sim N(\text{vec}(\Phi_0^*), Q \otimes \Omega_0) \end{aligned} \quad (15)$$

Following the Minnesota prior, the priors are set assuming parameters on longer lags are more likely to be zero. The diagonal elements of  $Q_0$ , denoted as  $q_{nn}$ , are set to the residual variances of the corresponding  $p$ -lag univariate autoregressions,  $\sigma_{nn}^2$  (Kadiyala and Karlsson, 1997). For the off-diagonal elements  $a_{nn}$ , where  $nn = 4, \dots, (K + M)$ ,<sup>13</sup> the prior mean is set to 0, with a large variance specified as  $A_0 = 10^3 \cdot I_{(K+M)}$ , ensuring that these elements are not restricted to zero.

$\Omega_0$  is an  $(1 + p(K + M)) \times (1 + p(K + M))$  matrix, with its diagonal elements set such that the prior variances of the parameter of  $l$  lagged  $mm^{th}$  variable in the  $nn^{th}$  equation equal  $\sigma_{nn}^2 / l\sigma_{mm}^2$ . Also, we set  $\Phi_0^* = 0$ .

The posterior is given as:

$$\begin{aligned} \gamma_{nn}|\tilde{X}_t, \tilde{\xi}_t &\overset{Ind}{\sim} IG\left(\frac{1+nn+T}{2}, \frac{\bar{q}_{nn}}{2}\right) \\ a_{nn}|\tilde{X}_t, \tilde{\xi}_t, \gamma_{nn} &\overset{Ind}{\sim} N(\bar{a}_{nn}, \bar{A}_{nn}) \\ \text{vec}(\Phi^*)|\tilde{X}_t, \tilde{\xi}_t, Q &\sim N(\text{vec}(\bar{\Phi}^*), Q \otimes \bar{\Omega}) \end{aligned} \quad (16)$$

<sup>13</sup>We require  $q_{21} = 0$ ,  $q_{31} = 0$  and  $q_{32} = 0$ , which implies setting  $a_{21} = 0$ ,  $a_{31} = 0$  and  $a_{32} = 0$  with probability 1. See Chan and Jeliazkov (2009) for additional details.



where  $\bar{q}_{nn}$  is the  $nn^{th}$  diagonal element of  $\bar{Q}$ .

$$\begin{aligned}
\bar{Q} &= Q_0 + \hat{v}'\hat{v} + (\hat{\Phi}^* - \Phi_0^*)' \left[ \Omega_0 + \left( \tilde{\xi}'_{T-1} \tilde{\xi}_{T-1} \right)^{-1} \right]^{-1} (\hat{\Phi}^* - \Phi_0^*) \\
\bar{A}_{nn} &= \gamma_{nn} (A_0^{-1} + \hat{v}'_{nn} \hat{v}_{nn})^{-1} \\
\bar{a}_{nn} &= \bar{A}_{nn} \left( \gamma_{nn}^{-1} \hat{v}'_{nn} \hat{V}_{nn} \right) \\
\hat{v}_{nn} &= [\hat{V}_1 : \dots : \hat{V}_{nn}] \\
\hat{V}_{nn} &= (\hat{V}_{1nn}, \hat{V}_{2nn}, \dots, \hat{V}_{Tnn})'
\end{aligned} \tag{17}$$

Also, we have,

$$\begin{aligned}
\bar{\Phi}^* &= \bar{\Omega} \left( \Omega_0^{-1} \Phi_0^* + \tilde{\xi}'_{T-1} \tilde{\xi}_{T-1} \hat{\Phi}^* \right) \\
\bar{\Omega} &= \left( \Omega_0^{-1} + \tilde{\xi}'_{T-1} \tilde{\xi}_{T-1} \right)^{-1}
\end{aligned} \tag{18}$$

Here,  $\hat{\Phi}^*$  and  $\hat{v}$  refer to the OLS estimates of the VAR coefficients and the reduced-form residuals for the first  $(K + M)$  equations of the VAR model. We discard draws of  $\Phi$  with roots outside the unit circle, as the VAR model would be unstable.

The process of drawing the factors, followed by drawing the parameters, constitutes one iteration in the multi-move Gibbs sampling. We preserve the  $\theta = (\Lambda, R^X, vec(\Phi), Q^\xi)$  estimates from the latter half of the 30,000 iterations, after discarding the initial 15,000 iterations.

## C Results (Ordering - Smallest to Largest)

We now comprehend the economic effects of an unexpected decrease in the share of the smallest component (federal non-defense investment) and the resulting adjustments in the other shares.<sup>14</sup> Since the component shares are ordered in ascending magnitude, the differences between successive loadings on  $f_2$  increase to reflect convexity.

### Responses to a Level Shock

Figure C1.1 displays the median impulse responses of the two latent factors—reallocation and curvature—alongside total government spending ( $G$ ), the fiscal deficit, and GDP, following a shock to total government spending ( $G$ ), with 16%–84% credible bands. The responses with shares in ascending order are quantitatively similar to those with descending order in the previous section.<sup>15</sup>

A shock to government spending causes the fiscal deficit and output to rise initially, with both peaking early and declining over time. The output multipliers for this level shock are presented in Table C1 and are identical to those reported in Table 1. These multipliers remain positive, diminishing gradually as output growth slows, with the 20-quarter horizon multiplier equal to 0.47.

### Responses of Shares to a Level Shock

Figure C1.3 presents the median impulse responses of the six component shares to a shock in  $G$ , along with the 16%–84% credible intervals. Similar to the responses in Figure 2.3, the individual shares exhibit no significant response over time. This lack of response is also evident in the cumulative shares, as illustrated in Figure C1.2. Consequently, the extracted factors, depicted in Figure C1.1, remain unaffected by the level shock.

Given the overall increase in total government spending, individual components also show an increase in spending, as shown in Figure C1.4. Each component increases by 0.2% on impact, corresponding to the response of  $G$  to its own shock. Larger components exhibit significant increases over longer horizons. Federal non-defense consumption peaks at 0.4% after four quarters before declining, with its increase remaining significant from the fourth to the fourteenth quarter. State and local government investment also peaks at 0.4%, maintaining

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<sup>14</sup>Instead of a unit shock, we scale the shocks by 1/4.

<sup>15</sup>The responses are scaled by 1/4, corresponding to the scaling of the shocks by 1/4.

significance from the third to the fourteenth quarter. Similarly, federal defense consumption rises by 0.3%, staying significant for up to fourteen quarters before gradually declining. With no significant changes in the proportion of spending across components, the multipliers in response to a level shock remain similar, regardless of whether the shares are arranged in descending or ascending order.

## Responses to a Reallocation Shock

Turning to Figure C2.1, a shock to the reallocation factor is transient, returning to the baseline within a quarter. This shock, however, leads to a significant, lasting decrease in the curvature factor, with  $f_2$  reaching its peak reduction after a quarter before gradually rising. Given that the curvature factor measures the relative significance of larger component shares, its decline implies an increase in smaller shares, a decrease in larger shares, or a mix of both. The response of government spending is insignificant.

The fiscal deficit increases by 0.12 percentage points on impact, with its response becoming insignificant after the first quarter. This initial fiscal strain is due to increased spending on non-defense consumption, while expenditures on federal non-defense investment contract. The GDP response to the reallocation shock remains negative throughout the forecast horizon, with statistical significance observed only initially. The initial significant decline in GDP of 0.05%, despite no immediate reduction in total  $G$ , reflects shifts in the distribution of spending across components.

The reallocation multiplier is positive, showing an upward trend across horizons, as shown in column (2) of Table C1. It is statistically significant only on impact, with a value of 0.83 over a 20-quarter horizon. A reallocation away from federal non-defense investment may not be beneficial for the economy.

## Responses of Shares to a Reallocation Shock

Figure C2.2 displays the median impulse responses of the five cumulative shares in reaction to a shock to  $f_1$ , with 16%–84% credible intervals. The first cumulative share,  $g_1$ , decreases by 2.5 percentage points on impact and returns to zero within one quarter, reflecting the inverse response of  $f_1$ .  $g_2$  decreases by 1.5 percentage points,  $g_3$  by around 1,  $g_4$  by roughly 0.6, and  $g_5$  by about 0.5 percentage points. This pattern indicates that component shares rise on impact, resulting in increasingly smaller declines across the cumulative shares. Furthermore,  $g_4$  and  $g_5$  show a more sustained decline, with  $g_4$  experiencing a drop of around 0.1 percentage

points and  $g_5$  around 0.2 percentage points. This persistence in the cumulative shares' decline contributes to the longer-lasting decline in the curvature factor seen in Figure C2.1.

As seen in Figure C2.3, component shares, other than  $s_1$ , increase in response to the reallocation shock on impact, as expected. Federal defense investment increases by about 1 percentage point on impact, while federal non-defense consumption and state and local government investment each rise by approximately 0.5 percentage points. These increases revert to their pre-shock levels after the first quarter. The significant decline in the curvature factor is largely due to reductions in the share of federal defense consumption. Defense consumption sees an initial rise of 0.2 percentage points, followed by a slightly larger long-lasting decline of 0.1 percentage points. Lastly, the share of state and local government consumption increases by 0.5 percentage points on impact, though this increase stabilizes at around 0.15 percentage points.

The spending responses in each component directly follow the changes in their shares. The decline in spending for state and local government investment and federal government consumption is greater than the increase observed in state and local government consumption, leading to an overall decrease in government spending following the reallocation shock.

The initial reduction in federal non-defense investment contributed to the 0.05% drop in GDP. Ellahie and Ricco (2017) reports that non-defense investment components generally have higher multipliers than non-defense consumption components, with the multiplier for federal non-defense investment exceeding 2 across horizons.

In summary, a reduction in the share of federal non-defense investment leads to an increase in the shares of all other components. Although the initial decline in the first component's share returns to zero after the first period, it prompts a reallocation of resources from federal government consumption and state and local government investment towards state and local government consumption. The impact of reallocation on output not only depends on which shares adjust in response to the decline of one but also on the magnitude of these changes.

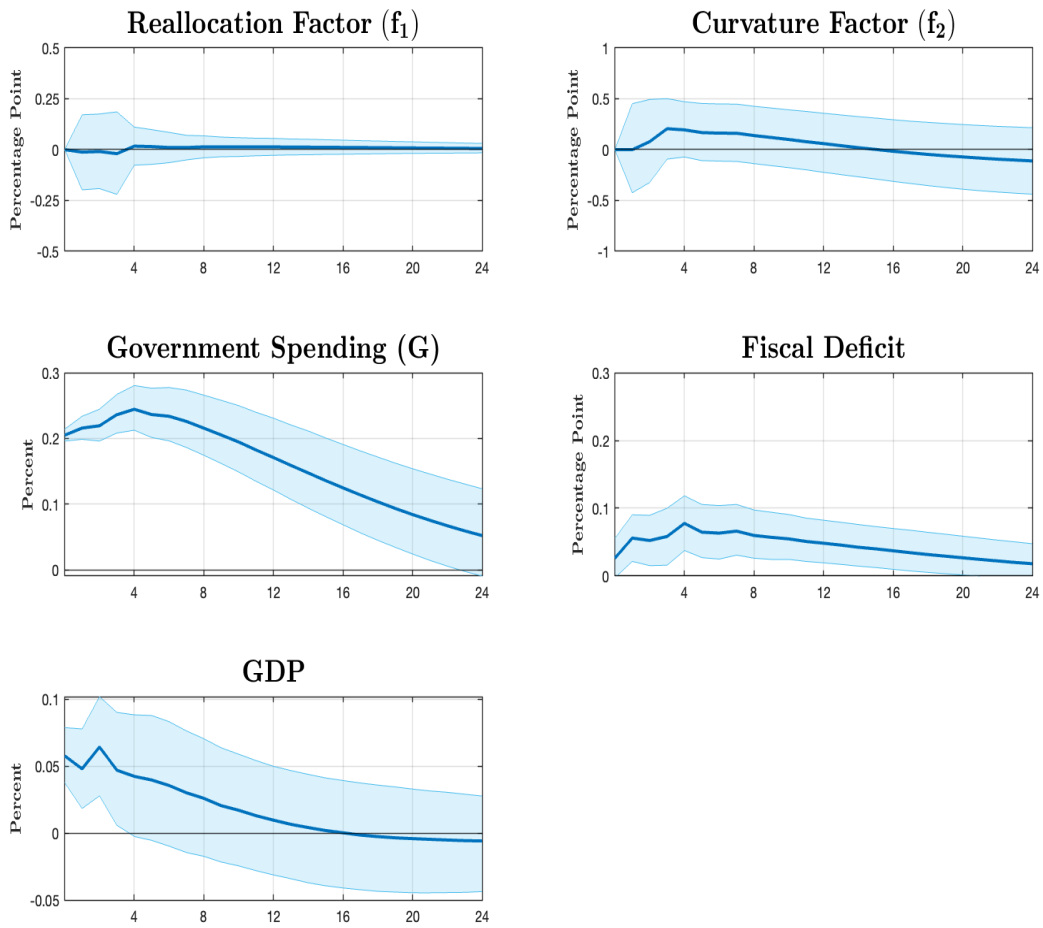


Figure C1.1: Impulse responses to a level shock (shock to  $G$ ) generated from FAVAR with two factors described by equations (2) and (3). Here, the level ( $G$ ) represents total government spending, which is the sum of all the six components considered. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

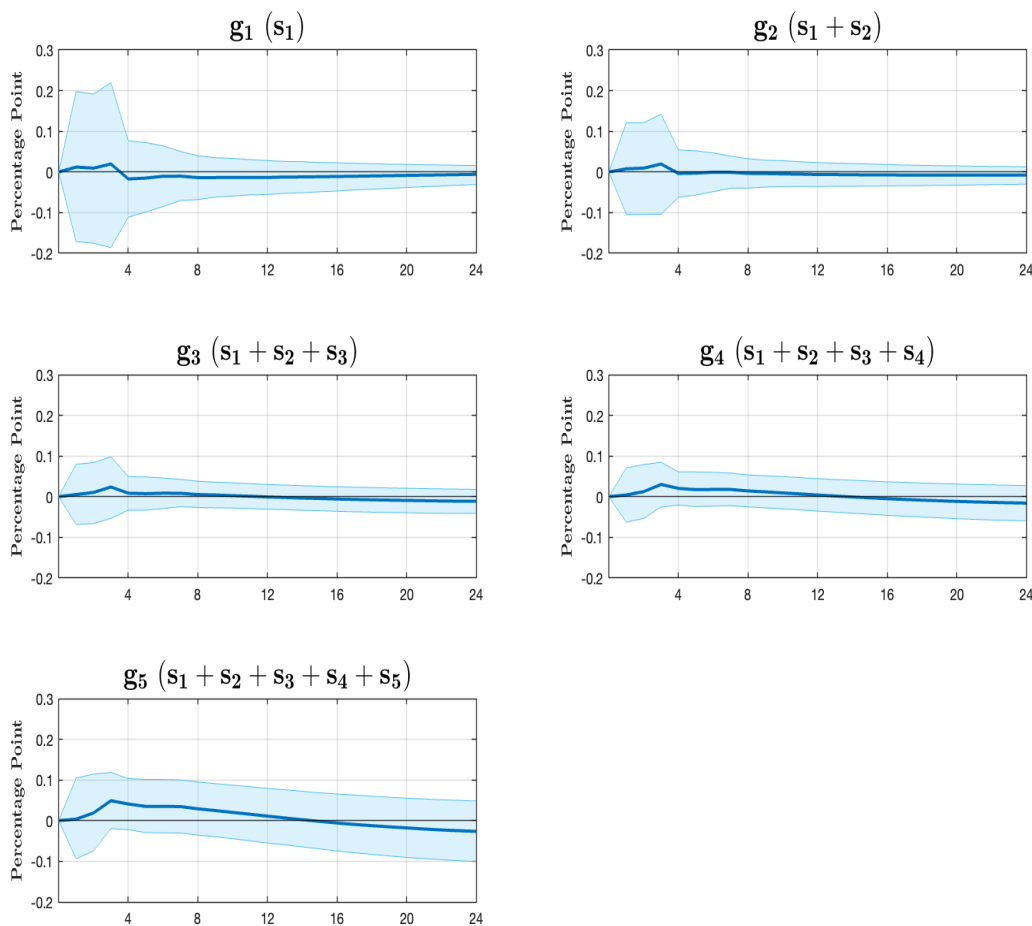


Figure C1.2: Impulse responses of cumulative shares ( $g_i$ ) to a level shock (shock to  $G$ ) generated from FAVAR with two factors described by equations (2) and (3). Here, the six individual shares arranged in ascending order are defined as follows - (1)  $s_1 = NDEFI/G$ , (2)  $s_2 = DEFI/G$ , (3)  $s_3 = NDEFC/G$ , (4)  $s_4 = SLGI/G$ , (5)  $s_5 = DEFC/G$  and (6)  $s_6 = SLGC/G$ . Here,  $G$  represents total government spending;  $NDEFI$  represents Federal Non-Defense Investment,  $DEFI$  denotes Federal Defense Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $SLGI$  is State and Local Government Investment,  $DEFC$  is Federal Defense Consumption, and  $SLGC$  is State and Local Government Consumption. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

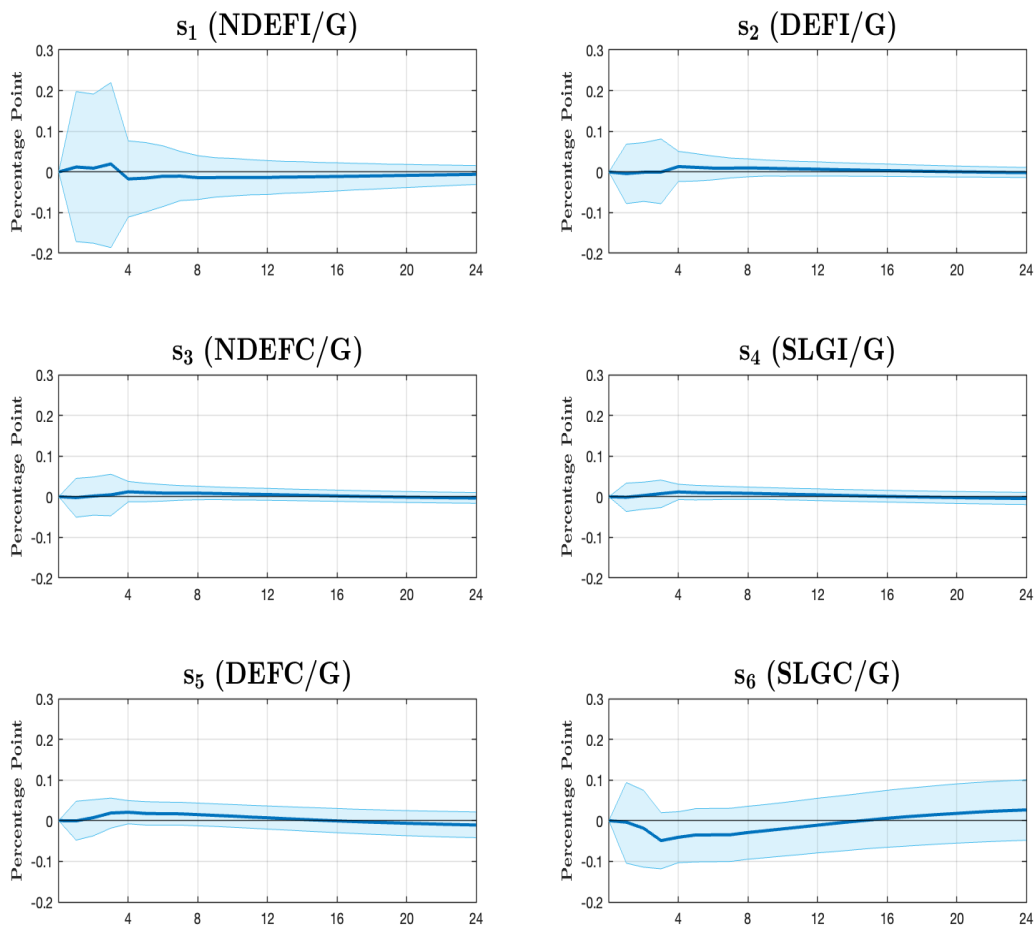


Figure C1.3: Impulse responses of individual shares ( $s_i$ ) to a level shock (shock to  $G$ ) generated from FAVAR with two factors described by equations (2) and (3). Here,  $G$  represents total government spending;  $NDEFI$  represents Federal Non-Defense Investment,  $DEFI$  denotes Federal Defense Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $SLGI$  is State and Local Government Investment,  $DEFC$  is Federal Defense Consumption, and  $SLGC$  is State and Local Government Consumption. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

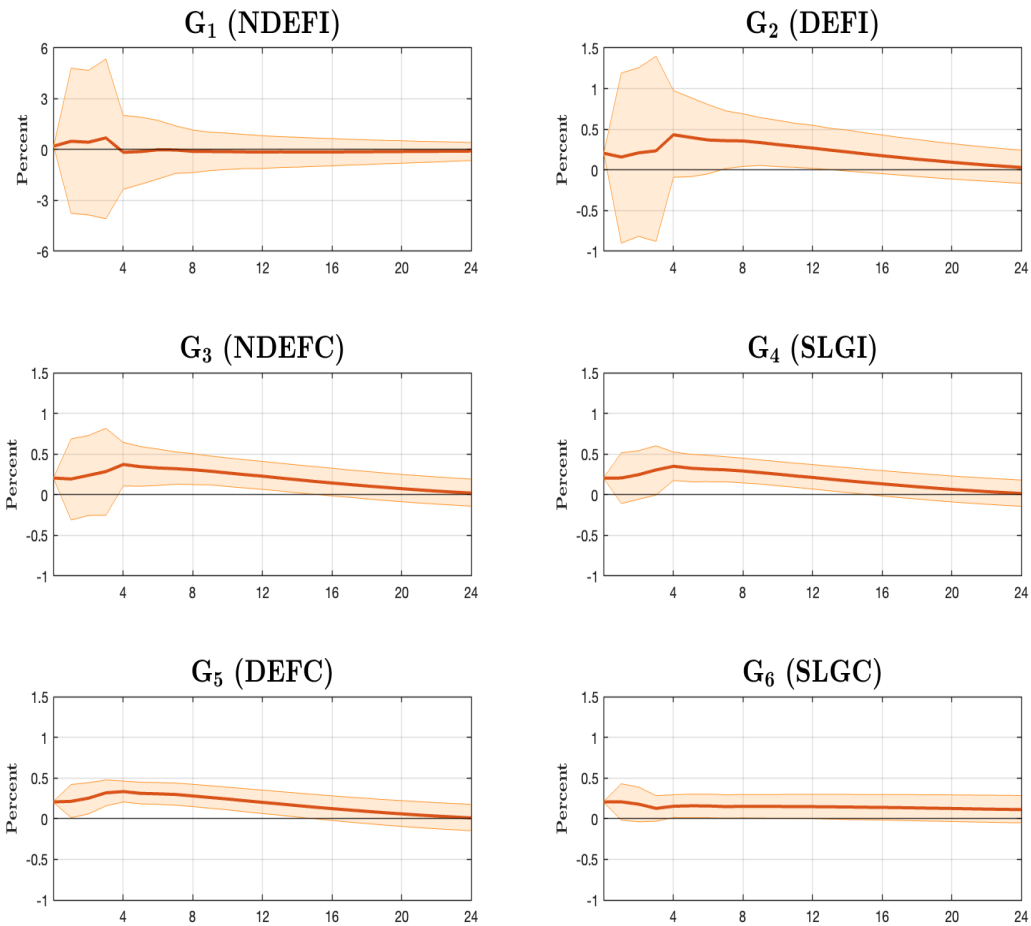


Figure C1.4: Impulse responses of spending on components ( $G_i$ ) to a level shock (shock to  $G$ ) generated from FAVAR with two factors described by equations (2) and (3). Here,  $G$  represents total government spending,  $NDEFI$  is Federal Non-Defense Investment,  $DEFI$  is Federal Defense Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $SLGI$  is State and Local Government Investment,  $DEFC$  is Federal Defense Consumption, and  $SLGC$  is State and Local Government Consumption. The individual shares are arranged in an ascending order. Orange solid lines represent medians, and the shaded area represents the 68% credible bands.



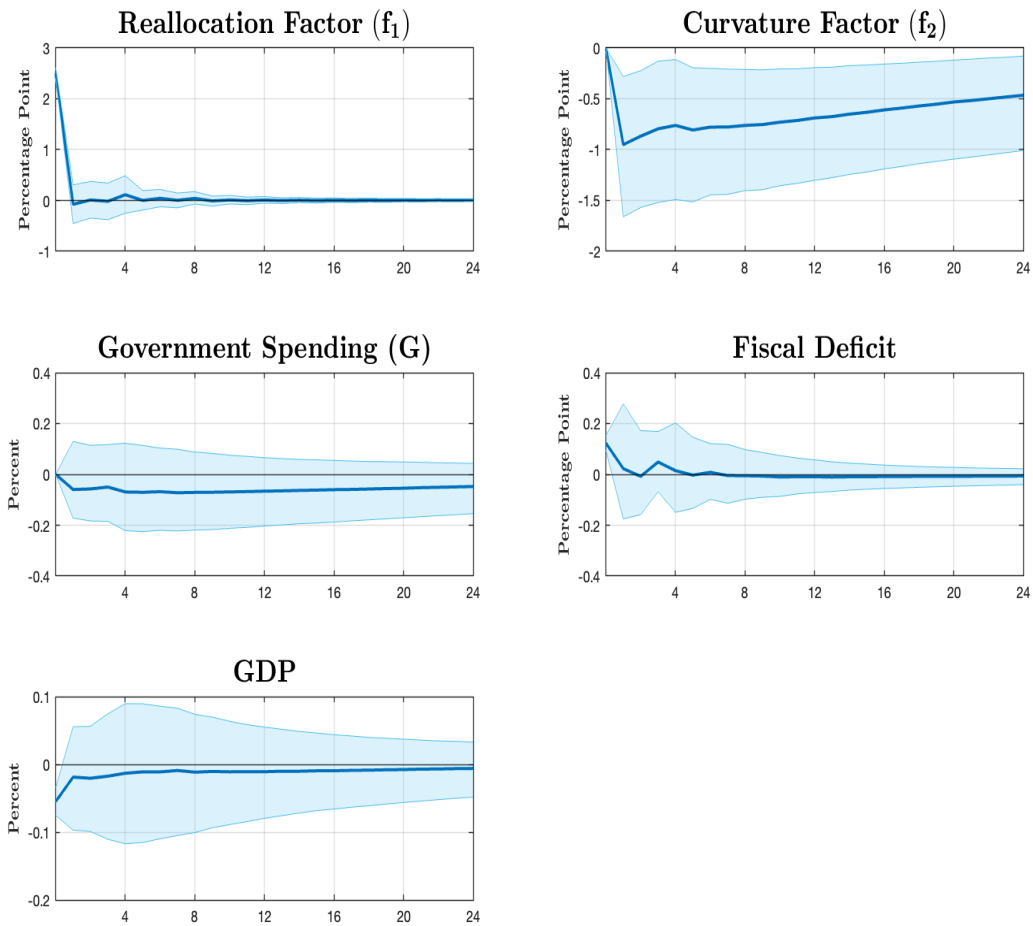


Figure C2.1: Impulse responses to a reallocation shock (shock to  $f_1$ ) generated from FAVAR with two factors described by equations (2) and (3). Here, the level ( $G$ ) represents total government spending, which is the sum of all the six components considered. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

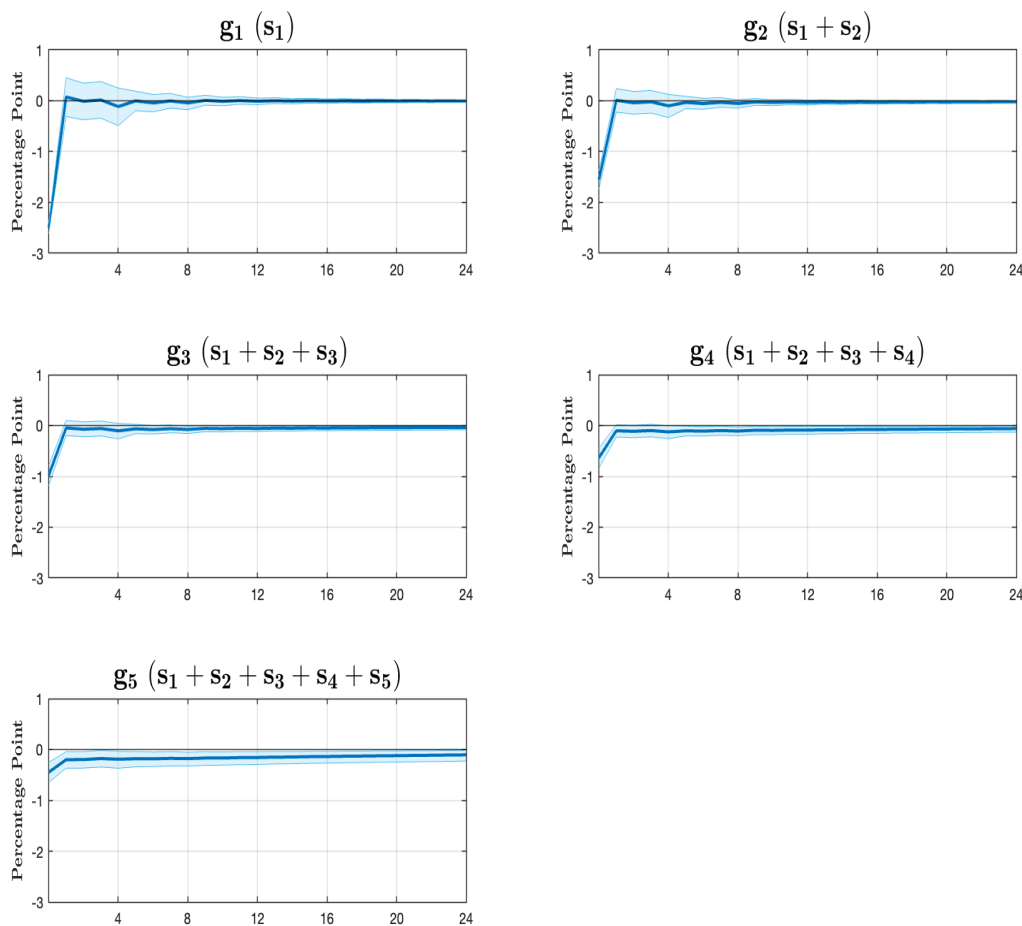


Figure C2.2: Impulse responses of cumulative shares ( $g_i$ ) to aa reallocation shock (shock to  $f_1$ ) generated from FAVAR with two factors described by equations (2) and (3). Here, the six individual shares arranged in ascending order are defined as follows - (1)  $s_1 = NDEFI/G$ , (2)  $s_2 = DEFI/G$ , (3)  $s_3 = NDEFC/G$ , (4)  $s_4 = SLGI/G$ , (5)  $s_5 = DEFC/G$  and (6)  $s_6 = SLGC/G$ . Here,  $G$  represents total government spending;  $NDEFI$  represents Federal Non-Defense Investment,  $DEFI$  denotes Federal Defense Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $SLGI$  is State and Local Government Investment,  $DEFC$  is Federal Defense Consumption, and  $SLGC$  is State and Local Government Consumption. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

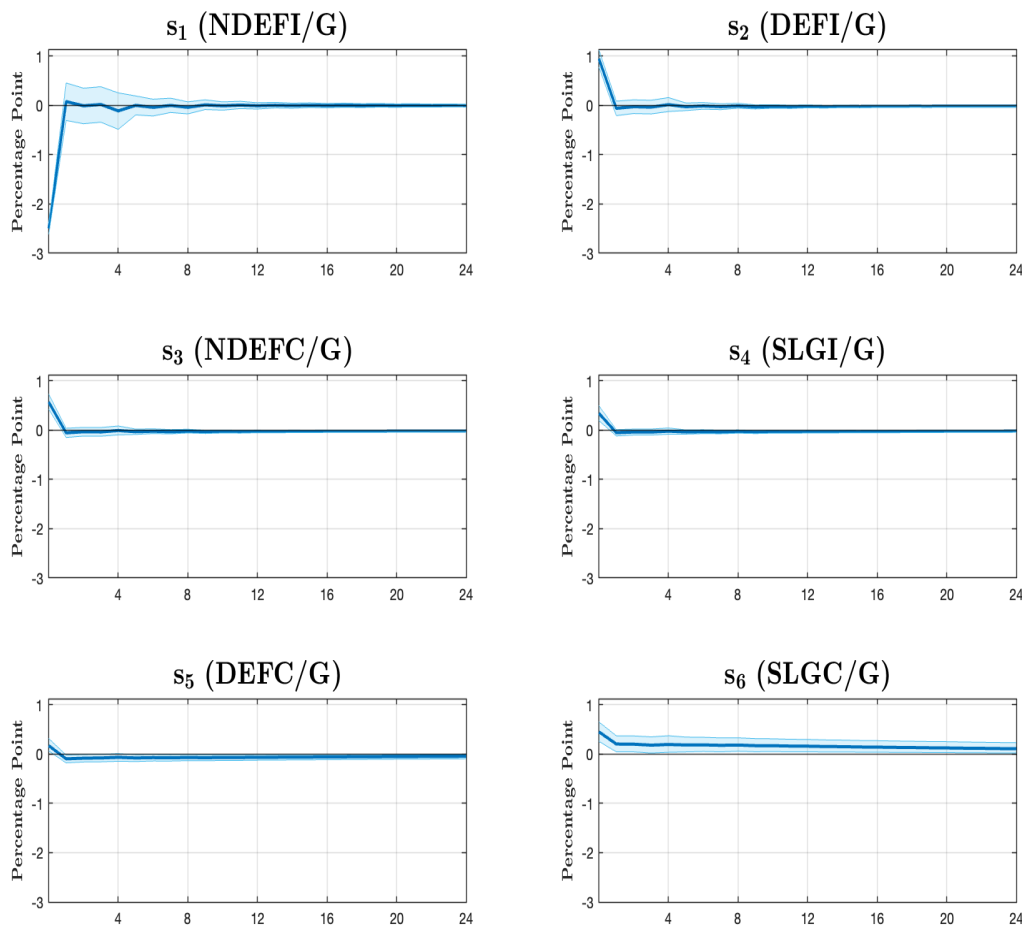


Figure C2.3: Impulse responses of individual shares ( $s_i$ ) to a reallocation shock (shock to  $f_1$ ) generated from FAVAR with two factors described by equations (2) and (3). Here,  $G$  represents total government spending;  $NDEFI$  represents Federal Non-Defense Investment,  $DEFI$  denotes Federal Defense Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $SLGI$  is State and Local Government Investment,  $DEFC$  is Federal Defense Consumption, and  $SLGC$  is State and Local Government Consumption. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

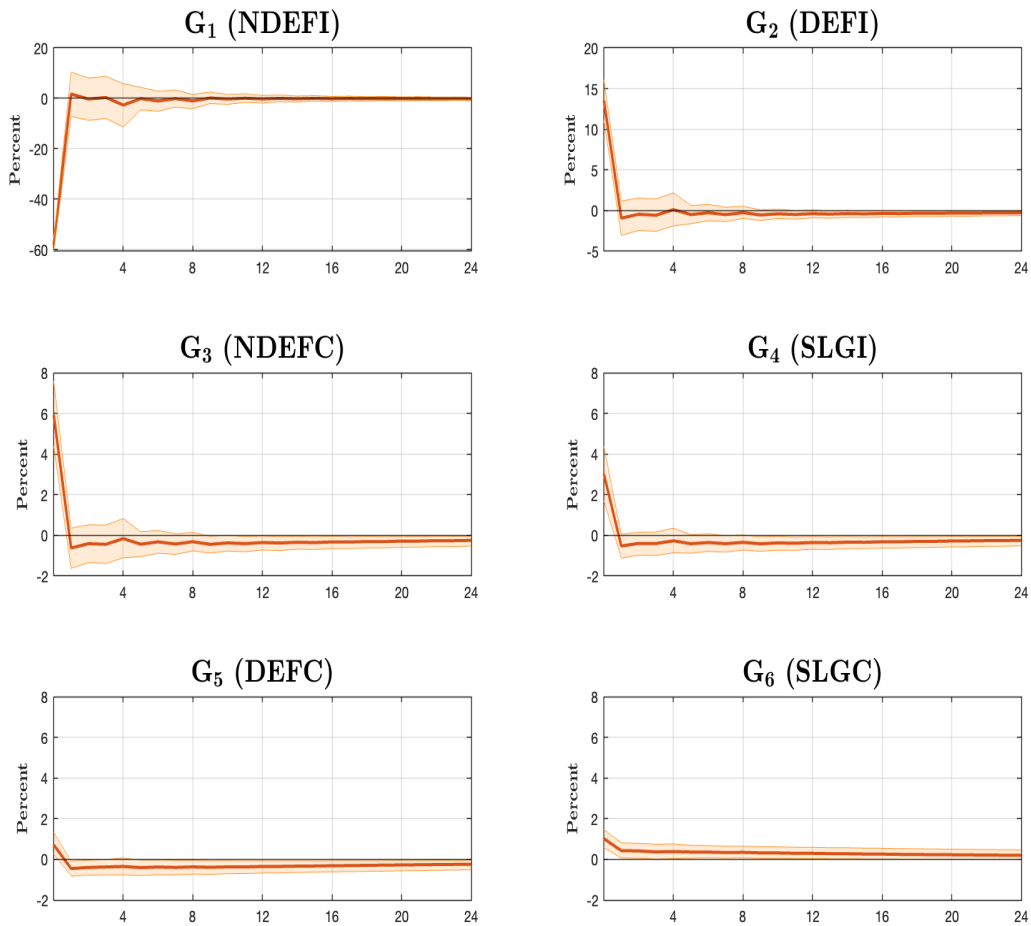


Figure C2.4: Impulse responses of spending on components ( $G_i$ ) to a reallocation shock (shock to  $f_1$ ) generated from FAVAR with two factors described by equations (2) and (3). Here,  $G$  represents total government spending,  $NDEFI$  is Federal Non-Defense Investment,  $DEFI$  is Federal Defense Investment,  $NDEFC$  is Federal Non-Defense Consumption,  $SLGI$  is State and Local Government Investment,  $DEFC$  is Federal Defense Consumption, and  $SLGC$  is State and Local Government Consumption. The individual shares are arranged in an ascending order. Orange solid lines represent medians, and the shaded area represents the 68% credible bands.

Table C1: Discounted Cumulative Multipliers

This table presents the discounted cumulative multipliers for GDP across the full sample at different horizons with 68% credible bands in brackets. Columns (1) and (2) represent discounted multipliers in response to a level shock (shock to  $G$ ) and a reallocation shock (shock to  $f_1$ ) when the individual shares are arranged in ascending order. Column (3) represents discounted multipliers in response to a composite shock to the component (NDEFI).

Horizon	Level multiplier (1)	Reallocation multiplier (2)
On Impact	1.40 [0.91, 1.91]	0.11 [0.07, 0.15]
1 quarters	1.25 [0.70, 1.79]	0.16 [-0.03, 0.35]
4 quarters	1.14 [0.46, 1.82]	0.27 [-0.45, 1.02]
8 quarters	0.93 [0.15, 1.68]	0.37 [-1.23, 2.05]
12 quarters	0.75 [-0.10, 1.49]	0.50 [-1.95, 3.17]
16 quarters	0.60 [-0.37, 1.36]	0.64 [-2.53, 4.25]
20 quarters	0.47 [-0.68, 1.28]	0.76 [-3.10, 5.35]
24 quarters	0.36 [-1.06, 1.26]	0.83 [-3.65, 6.23]

Table C2: Non-discounted Cumulative Multipliers

This table presents the cumulative multipliers for GDP across the full sample at different horizons with 68% credible bands in brackets. Columns (1) and (2) represent multipliers in response to a level shock (shock to  $G$ ) and a reallocation shock (shock to  $f_1$ ) when the individual shares are arranged in ascending order. Column (3) represents multipliers in response to a composite shock to the component (NDEFI).

Horizon	Level multiplier (1)	Reallocation multiplier (2)
On Impact	1.40 [0.91, 1.91]	0.11 [0.07, 0.15]
1 quarters	1.26 [0.71, 1.79]	0.15 [-0.02, 0.34]
4 quarters	1.15 [0.48, 1.82]	0.25 [-0.39, 0.92]
8 quarters	0.96 [0.20, 1.69]	0.33 [-0.95, 1.67]
12 quarters	0.81 [-0.01, 1.53]	0.41 [-1.39, 2.33]
16 quarters	0.69 [-0.21, 1.41]	0.49 [-1.67, 2.85]
20 quarters	0.59 [-0.38, 1.34]	0.56 [-1.86, 3.28]
24 quarters	0.52 [-0.57, 1.30]	0.60 [-2.01, 3.62]